Received: *March 20, 1998,* Accepted: *May 22, 1998*

Equations of the (2,0) theory and knitted fivebranes

Ori Ganor
Department of Physics, Princeton University,
Princeton, NJ 08544
E-mail: `origa@puhep1.princeton.edu`

Luboš Motl
Department of Physics and Astronomy, Rutgers University
Piscataway, NJ 08855-0849
E-mail: `motl@physics.rutgers.edu`

Abstract: We study non-linear corrections to the low-energy description of the (2,0) theory. We argue for the existence of a topological correction term similar to the $C_3 \wedge X_8(R)$ in M-theory. This term can be traced to a classical effect in supergravity and to a one-loop diagram of the effective 4+1D Super Yang-Mills. We study other terms which are related to it by supersymmetry and discuss the requirements on the subleading correction terms from M(atrix) theory. We also speculate on a possible fundamental formulation of the theory.

Keywords: Field Theories in Higher Dimensions, String Duality, Superstring Vacua, M(atrix) Theories.

JHEP05(1998)009

Contents

JHEP05(1998)009

1. Introduction

During the last two years a lot of attention has been devoted to the newly discovered 5+1D theories [1]. The version of these theories with (2,0) supersymmetry arises as a low-energy description of type-II B on an A_{N-1} singularity [1] or as the dual low-energy description of N coincident 5-branes in M-theory [2]. Part of the attention [3, 4, 5] is due to the rôle they play in compactified M(atrix) theory [6], part is because they provide testing grounds to M(atrix) theory ideas [7, 8, 9, 10, 11], and another part is because they shed light on non-perturbative phenomena in 3+1D gauge theories [1]. These theories are also very exciting on their own right. They lack any parameter which will allow a classical perturbative expansion (like the coupling constant of SYM). Thus, these theories have no classical limit (for finite N). the only possible classical expansion is a derivative expansion where the energy is the small parameter.

One of our goals will be to explore the low-energy description of the (2,0) theory. At low energies, and a generic point in moduli space the zeroth order approximation is N free tensor multiplets which contain the chiral anti-self-dual 2-forms. Since the theory contains chiral 2-forms it is more convenient to write down the low-energy equations of motion rather than the non-manifestly covariant Lagrangian (there is the other option of using the manifestly covariant formulation of [12, 13], but using the equations of motion will be sufficient for our purposes). These equations are to be interpreted à la Wilson, i.e. as quantum equations for operators but with a certain unspecified UV cutoff. The leading terms in the Wilsonian low-energy description are the linear equations of motion for the N free tensor multiplets. We will be looking for the first sub-leading corrections. Those corrections will be non-linear and are a consequence of the interacting nature of the full (2,0) theory. In general at high enough order in the derivative expansion the terms in the Wilsonian action are cutoff dependent. However, we will see that the first order corrections are independent of the cutoff. We will argue that the low-energy equations contain a topological term somewhat analogous to the subleading $C_3 \wedge X_8(R)$ term of M-theory [14, 15] and which describes a topological correction term to the anti-self-dual string current. We will then study the implications of supersymmetry.

The paper is organized as follows. Section 2 is a review of the (2,0) theory. In section 3 we derive the topological term from the supergravity limit of N 5-branes of M-theory. Our discussion will be an implementation of results described in [16]. In section 4 we discuss the implied correction terms after compactification to 3+1D, and we find related terms which are implied by supersymmetry. In section 5 we discuss the currents in 5+1D. Finally, in sections 6, 7 we speculate on a possible "deeper" meaning of these correction terms.

After completion of this paper, we received a message about related works [17] which studied the single 5-brane solution in supergravity. We are grateful to N.D. Lambert for the correspondence.

2. Review of the (2,0) theory

This section is a short review of some facts we will need about the (2,0) theory.

2.1. Realization

The $(2,0)_N$ theory is realized either as the low-energy decoupled degrees of freedom from an A_{N-1} singularity (for $N \geq 2$) of type IIB [1] or from the low-energy decoupled degrees of freedom of N 5-branes of M-theory [2]. This is a conformal 5+1D theory which is interacting for $N > 1$. It has a chiral (2,0) supersymmetry with 16 generators. One can deform the theory away from the conformal point. This corresponds to separating the N 5-branes (or blowing up the A_{N-1} singularity). If the separation scale x is much smaller than the 11D Planck length M_p^{-1} then at energies $E \sim M_p^{3/2} x^{1/2}$ one finds a massive decoupled theory whose low-energy description is given by N free tensor multiplets.

Each free tensor multiplet in 5+1D comprises of 5 scalar fields Φ^A with $A = 1 \dots 5$, one tensor field $B_{\mu\nu}^{(-)}$ where the $(-)$ indicates that its equations of motion force it to be anti-self-dual, and 4 multiplets of chiral fermions Θ. The (2,0) supersymmetry in 5+1D has $Sp(2) = Spin(5)_R$ R-symmetry. The scalars Φ^A are in the $\underline{\mathbf{5}}$ whereas the fermions are in the $(\underline{\mathbf{4}}, \underline{\mathbf{4}})$ of $SO(5,1) \times Sp(2)$ but with a reality condition. Thus there are 16 real fields in Θ.

For the low-energy of the $(2,0)_N$ theory there are N such tensor multiplets. The moduli space, however, is not just $(\mathbb{R}^5)^N$ because there are discrete identifications given by the permutation group. It is in fact $(\mathbb{R}^5)^N / S_N$. Let us discuss what happens for $N = 2$. The moduli space can be written as $\mathbb{R}^5 \times (\mathbb{R}^5/\mathbb{Z}_2)$. The first $\mathbb{R}^5$ is the sum of the two tensor multiplets. In 5+1D this sum is described by a free tensor multiplet which decouples from the rest of the theory (although after compactification, it has some global effects which do not decouple). The remaining $\mathbb{R}^5/\mathbb{Z}_2$ is the difference of the two tensor multiplets. This moduli space has a singularity at the origin where the low-energy description is no longer two free tensor multiplets but is the full conformal theory.

2.2. Equations of motion for a free tensor multiplet

To write down the lowest order equations of motion for a free tensor multiplet we use the field strength

$$H_{\alpha\beta\gamma} = 3\partial_{[\alpha} B_{\beta\gamma]}^{(-)} .$$

This equation does not imply that H is anti-self-dual but does imply that H is a closed form. It is possible to modify this equation such that H will be manifestly anti-self-dual. We will define H to be anti-self-dual part of dB according to,

$$H_{\alpha\beta\gamma} = \frac{3}{2}(\partial_{[\alpha} B_{\beta\gamma]}) - \frac{1}{4}\epsilon_{\alpha\beta\gamma}{}^{\alpha'\beta'\gamma'}(\partial_{\alpha'} B_{\beta'\gamma'}) . \tag{2.1}$$

JHEP05(1998)009

This definition is the same as the previous one for anti-self-dual dB, it trivially implies that H is anti-self-dual and it does not lead to the equation $dH = 0$ which we will find useful later on. In any case, we will use the equations of motion for H only and B will therefore not appear. For the fermions it is convenient to use 11D Dirac matrices

$$\Gamma^\mu,\ \mu = 0\dots 5\,, \qquad \Gamma^A,\ A = 6\dots 10\,,$$

with commutation relations

$$\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}\,, \qquad \{\Gamma^A, \Gamma^B\} = 2\delta^{AB}\,, \qquad \{\Gamma^A, \Gamma^\mu\} = 0\,.$$

We define

$$\widetilde{\Gamma} = \Gamma^{012345} = \Gamma^0\Gamma^1\cdots\Gamma^5 = \Gamma^6\Gamma^7\cdots\Gamma^{10}\,.$$

The spinors have positive chirality and satisfy

$$\Theta = \widetilde{\Gamma}\Theta\,.$$

The $Spin(5)_R$ acts on Γ^A while $SO(5,1)$ acts on Γ^μ. The free equations of motion are given by

$$\begin{aligned} H^{\mu\nu\sigma} &= \frac{1}{6}\epsilon_{\tau\rho\gamma}{}^{\mu\nu\sigma}H^{\tau\rho\gamma} \equiv -\frac{1}{6}\epsilon^{\mu\nu\sigma}{}_{\tau\rho\gamma}H^{\tau\rho\gamma}\,, &(2.2)\\ \partial_{[\tau}H_{\mu\nu\sigma]} &= 0, &(2.3)\\ \Box\Phi^A &= 0\,, &(2.4)\\ \partial\!\!\!/\Theta &= 0\,. &(2.5)\end{aligned}$$

The supersymmetry variation is given by

$$\begin{aligned} \delta H_{\alpha\beta\gamma} &= -\frac{i}{2}\bar{\epsilon}\Gamma_\delta\Gamma_{\alpha\beta\gamma}\partial^\delta\Theta &(2.6)\\ \delta\Phi_A &= -i\bar{\epsilon}\Gamma_A\Theta &(2.7)\\ \delta\Theta &= (\frac{1}{12}H_{\alpha\beta\gamma}\Gamma^{\alpha\beta\gamma} + \Gamma^\alpha\partial_\alpha\Phi_A\Gamma^A)\epsilon\,. &(2.8)\end{aligned}$$

The quantization of the theory is slightly tricky. There is no problem with the fermions Θ and bosons Φ^A, but the tensor field is self-dual and thus has to be quantized similarly to a chiral boson in 1+1D. This means that we second-quantize a free tensor field without any self-duality constraints and then set to zero all the oscillators with self-dual polarizations. The action that we use in 5+1D is:

$$\mathcal{A} = -\frac{1}{4\pi}\int\left\{\partial_\mu\Phi^A\partial^\mu\Phi_A + \frac{3}{2}\partial_{[\mu}B_{\sigma\tau]}\partial^{[\mu}B^{\sigma\tau]} + i\bar{\Theta}\,\partial\!\!\!/\Theta\right\}d^6\sigma\,.$$

Here we have defined $\bar{\Theta} = \Theta^T\Gamma^0$. The normalization is such that integrals of $B_{\sigma\tau}$ over closed 2-cycles live on circles of circumference 2π. In appendix A we list some more useful formulas.

3. Low-energy correction terms – derivation from SUGRA

In this section we will derive a correction term to the zeroth order low-energy terms.

Let us consider two 5-branes in M-theory. Let their center of mass be fixed. The fluctuations of the center of mass are described by a free tensor multiplet. Let us assume that the distance between the 5-branes at infinity $|M_p^{-2}\Phi_0|$ is much larger than the 10+1D Planck length M_p^{-1} and let us consider the low-energy description of the system for energies $E \ll |\Phi_0|$. The description at lowest order is given by supergravity in the 10+1D bulk and by a 5+1D tensor multiplet with moduli space $\mathbb{R}^5/\mathbb{Z}_2$ (we neglect the free tensor multiplet coming from the overall center of mass). The lowest order equations of motion for the tensor multiplet are the same linear equations as described in the previous section. We would like to ask what are the leading nonlinear corrections to the linear equations.

We will now argue that according to the arguments given in [16] there is a topological contribution to the dH equation of motion (here $\Phi^{(ij)} \equiv \Phi^{(i)} - \Phi^{(j)}$)

$$\partial_{[\alpha} H^{(i)}_{\beta\gamma\delta]} = \sum_{j=1\dots N}^{(j\neq i)} \frac{3\epsilon^{ABCDE}}{16\pi|\Phi^{(ij)}|^5} \Phi^{E,(ij)} \partial_{[\alpha}\Phi^{A,(ij)} \partial_\beta \Phi^{B,(ij)} \partial_\gamma \Phi^{C,(ij)} \partial_{\delta]} \Phi^{D,(ij)} \,. \tag{3.1}$$

Here $A \dots E = 1 \dots 5$. Φ^A are the scalars of the tensor multiplet and $H_{\alpha\beta\gamma}$ is the anti-self-dual field strength. Note that the RHS can be written as a pullback $\pi^*\omega_4$ of a closed form on the moduli space which is

$$\mathcal{M} \equiv \mathbb{R}^5/\mathbb{Z}_2 - \{0\} \,.$$

Here

$$\pi : \mathbb{R}^{5,1} \longrightarrow \mathcal{M} = \mathbb{R}^5/\mathbb{Z}_2 - \{0\}$$

is the map Φ^A from space-time to the moduli space and

$$\omega_4 = \frac{3}{8\pi^2|\Phi|^5} \epsilon^{ABCDE} \Phi^E d\Phi^A \wedge d\Phi^B \wedge d\Phi^C \wedge d\Phi^D$$

is half an integral form in $H_4(\frac{1}{2}\mathbb{Z})$, i.e.

$$\int_{S^4/Z_2} \omega_4 = \frac{1}{2} \,.$$

Let us explain how (3.1) arises. When Φ^A changes smoothly and slowly, the supergravity picture is that each 5-brane "wraps" the other one. Each 5-brane is a source for the (dual of the) $F_4 = dC_3$ 4-form field-strength of 10+1D supergravity. When integrated on a sphere S^4 surrounding the 5-brane we get $\int_{S^4} F_4 = 2\pi$. The other 5-brane now feels an effective C_3 flux on its world-volume. This, in turn, is a source for the 3-form anti-self-dual low-energy field-strength $dH = dC_3$. It follows that the total string charge measured at infinity of the $\mathbb{R}^{5,1}$ world-volume of one 5-brane is,

$$\int dH = \int dC_3 = \int F_4 \,.$$

JHEP05(1998)009

The integrals here are on $\mathbb{R}^4$ which is a subspace of $\mathbb{R}^{5,1}$ and they measure how much effective string charge passes through that $\mathbb{R}^4$. The integral on the RHS can now be calculated. It is the 4D-angle subtended by the $\mathbb{R}^4$ relative to the second 5-brane which was the source of the F_4. But this angle can be expressed solely in terms of Φ^A and the result is the integral over ω_4.

These equations can easily be generalized to N 5-branes. We have to supplement each field with an index $i = 1 \ldots N$. We can also argue that there is a correction

$$\Box \Phi^{D,(i)} = - \sum_{j=1\ldots N}^{(j\neq i)} \frac{\epsilon^{ABCDE}}{32\pi |\Phi^{(ij)}|^5} \Phi^{E,(ij)} \partial_\alpha \Phi^{A,(ij)} \partial_\beta \Phi^{B,(ij)} \partial_\gamma \Phi^{C,(ij)} H^{\alpha\beta\gamma,(ij)} + \cdots \tag{3.2}$$

Here $\Phi^{(ij)} \equiv \Phi^{(i)} - \Phi^{(j)}$ and similarly $H^{(ij)} = H^{(i)} - H^{(j)}$. The term $(\cdots)$ contains fermions and other contributions.

The equation (3.2) for $\Box\Phi$ can be understood as the equation for force between a tilted fivebrane and another fivebrane which carries an $H_{\alpha\beta\gamma}$ flux. As far as BPS charges go, the H flux inside a 5-brane is identified in M-theory with a membrane flux. This means that (after compactification) as a result of a scattering of a membrane on a 5-brane an H-flux can be created and the membrane can be annihilated. The identification of the H-flux with the membrane charge is also what allows a membrane to end on a 5-brane [2]. Consistency implies that a 5-brane with an H flux should exert the same force on other objects as a 5-brane and a membrane. This is indeed the case, as follows from the $C_3 \wedge H$ interaction on the 5-brane world-volume [2].

The Lorentz force acting on a point like particle equals (in its rest frame)

$$m \frac{d^2}{dt^2} x^i = e \cdot F^{0i} \,. \tag{3.3}$$

As a generalization for a force acting on the fivebrane because of the flux H in the other 5-brane, we can replace d^2/dt^2 by $\Box$ and write

$$\Box \Phi_A = F_{A\alpha\beta\gamma} H^{\alpha\beta\gamma} \,. \tag{3.4}$$

But we must calculate the four-form supergravity field strength at the given point. Only components with one Latin index and three Greek indices are important. We note that the electric field strength in the real physical 3+1-dimensional electrostatics is proportional to

$$F_{0A} \propto \frac{r_A}{r^3} \propto \frac{1}{r^2} \,. \tag{3.5}$$

The power 3 denotes 3 transverse directions, F contains all the indices in which the "worldvolume" of the particle is stretched. As an analogue for fivebrane stretched exactly in 012345 directions,

$$*F_{012345A} \propto \frac{\Phi_A^{(ij)}}{|\Phi_{(ij)}|^5} \propto \frac{1}{|\Phi_{(ij)}|^4} \,. \tag{3.6}$$

We wrote star because we interpret the fivebrane as the "magnetic" source. F in (3.4) has one Latin index and three Greek indices, so its Hodge dual has four Latin indices and three Greek indices. $*F$ in (3.6) contains only one Latin index but when the 5-branes are tilted by infinitesimal angles $\partial_\gamma \Phi_C$ we get also a contribution to the desired component of F:

$$*F_{\alpha\beta\gamma ABCD} = *F_{\alpha\beta\gamma\delta\sigma\tau D}\partial^\delta \Phi_A^{(ij)}\partial^\sigma \Phi_B^{(ij)}\partial^\tau \Phi_C^{(ij)} . \tag{3.7}$$

Now if we substitute (3.6) to (3.7) and the result insert to (3.4), we get the desired form of the $\Box\Phi$ equations.

Similarly, there is an equation for Θ,

$$\not{\partial}\Theta^i \propto \sum_{j=1\dots N}^{(j\neq i)} \frac{\epsilon^{ABCDE}}{|\Phi^{(ij)}|^5}(\Phi^{(ij)})_E \partial_\alpha(\Phi^{(ij)})_A \partial_\beta(\Phi^{(ij)})_B \partial_\gamma(\Phi^{(ij)})_C \Gamma^{\alpha\beta\gamma}\Gamma_D \Theta^{(ij)} . \tag{3.8}$$

Our goal in this paper is to deduce the corrections in the derivative expansion in the low-energy of the (2,0) theory. We cannot automatically deduce that (3.1), (3.2) and (3.8) can be extrapolated to the (2,0) theory because this description is valid only in the opposite limit, when $|\Phi| \ll M_p$, and supergravity is not a good approximation. However, the RHS of (3.1) is a closed 4-form on the moduli space $\mathcal{M} = \mathbb{R}^5/\mathbb{Z}_2 - \{0\}$ which is also half integral, i.e. in $H_4(\mathcal{M}, \frac{1}{2}\mathbb{Z})$. It must remain half-integral as we make $|\Phi|$ smaller. Otherwise, Dirac quantization will be violated. (Note that the wrapping number is always even.) Eqn. (3.2) follows from the same term in the action as (3.1). As for other correction terms, if we can show that they are implied by (3.1) and supersymmetry, then we can trust them as well. This will be the subject of the next section.

We would like to point out that this reasoning is somewhat similar to that of [42, 41] who related the R^4 terms in 11D M-theory to the $C\wedge X_8(R)$ term of [14, 15].

4. Compactification

In this section we will study the reduction of the terms to 3+1D by compactifying on $\mathbf{T}^2$. Let A be the area of $\mathbf{T}^2$ and τ be its complex structure. At low-energy in 3+1D we obtain a free vector multiplet of $N = 4$ with coupling constant τ. We are interested in the subleading corrections to the Wilsonian action. We will study these corrections as a function of A. Let us first note a few facts (see [18] for a detailed discussion).

When one reduces classically a free tensor multiplet from 5+1D down to 3+1D one obtains a free vector-multiplet with one photon and 6 scalars. Out of the 6 scalars one is compact. This is the scalar that was obtained from B_{45}. We denote it by σ.

$$\sigma = (\mathrm{Im}\tau)^{-1/2} A^{-1/2} \int_{\mathbf{T}^2} B_{45} .$$

We have normalized its kinetic energy so as to have an $\mathrm{Im}\tau$ in front, like 3+1D SYM. The radius of σ is given by

$$\sigma \sim \sigma + 2\pi(\mathrm{Im}\tau)^{-1/2}A^{-1/2}\,. \tag{4.1}$$

In 5+1D there was a $Spin(5)_R$ global symmetry. $N = 4$ SYM has $Spin(6)_R$ global symmetry but the dimensional reduction of the (2,0)-theory has only $Spin(5)_R$. Let us also denote by Φ_0 the square root of sum of squares of the VEV of the 5 scalars other than σ.

Now let us discuss the interacting theory. When $\Phi_0 A \ll 1$ we can approximate the 3+1D theory at energy scales $E \ll A^{-1}$ by 3+1D SYM. In this case the $Spin(5)_R$ is enhanced, at low-energy, to $Spin(6)_R$. For $\Phi_0 A \gg 1$ the "dynamics" of the theory occurs at length scales well below the area of the $\mathbf{T}^2$ where the theory is effectively (5+1)-dimensional. The 3+1D low-energy is therefore the classical dimensional reduction of the 5+1D low-energy. Thus, from our 3+1D results below we will be able to read off the 5+1D effective low-energy in this regime.

4.1. Dimensional reduction of the correction term

Let us see what term we expect to see at low-energy in 3+1D. We take the term,

$$\partial_{[\alpha}H_{\beta\gamma\delta]} = \frac{3}{16\pi|\Phi|^5}\epsilon_{ABCDE}\Phi^E\partial_\alpha\Phi^A\partial_\beta\Phi^B\partial_\gamma\Phi^C\partial_\delta\Phi^D\,,$$

and substitute 0123 for $\alpha\beta\gamma\delta$. The field $H_{\beta\gamma\delta}$ is,

$$H_{\beta\gamma\delta} = -\epsilon_{\beta\gamma\delta}{}^{\alpha}H_{\alpha 45} = -(\mathrm{Im}\tau)^{1/2}A^{-1/2}\epsilon_{\beta\gamma\delta\alpha}\partial^\alpha\sigma\,.$$

The equation becomes

$$\partial^\mu\partial_\mu\sigma = -\frac{1}{32\pi}(\mathrm{Im}\tau)^{-1/2}A^{1/2}\frac{1}{|\Phi|^5}\epsilon_{ABCDE}\Phi^E\partial_\alpha\Phi^A\partial_\beta\Phi^B\partial_\gamma\Phi^C\partial_\delta\Phi^D\epsilon^{\alpha\beta\gamma\delta}\,.$$

Here $\Phi^A\ldots\Phi^E$ are the six-dimensional fields. The 4-dimensional fields are defined by

$$\Phi^A = (\mathrm{Im}\tau)^{1/2}A^{-1/2}\varphi^A\,. \tag{4.2}$$

Thus, the action should contain a piece of the form

$$\begin{aligned}&\frac{1}{32\pi}(\mathrm{Im}\tau)\int d^4x\ \partial_\mu\sigma\partial^\mu\sigma - \\ &\qquad -\frac{1}{32\pi}(\mathrm{Im}\tau)^{1/2}A^{1/2}\epsilon_{ABCDE}\int d^4x\ \frac{\sigma}{|\varphi|^5}\epsilon^{\alpha\beta\gamma\delta}\varphi^E\partial_\alpha\varphi^A\partial_\beta\varphi^B\partial_\gamma\varphi^C\partial_\delta\varphi^D\,.\end{aligned} \tag{4.3}$$

Note that this is the behavior we expect when $\Phi_0 A \gg 1$. When $\Phi_0 A \sim 1$ the approximation of reducing the 5+1D effective action is no longer valid as explained above.

Let us first see how to write such a term in an $N = 1$ superfield notation. Let us take three chiral superfields, Φ and Φ^I ($I = 1, 2$). We assume that

$$\Phi = \varphi_0 + \delta\varphi + i\sigma\,.$$

σ is the imaginary part of Φ and φ_0 is the VEV of the real part. Below, the index I of Φ^I is lowered and raised with the anti-symmetric ϵ_{IJ}.

4.2. Interpolation between 3+1D and 5+1D

In the previous section we assumed that we are in the region $\Phi_0 A \gg 1$. This was the region where classical dimensional reduction from 5+1D to 3+1D is a good approximation. However, the question that we are asking about the low-energy effective action makes sense for any A. For $\Phi_0 A \sim 1$ quantum effects are strong. Let us concentrate on another possible term which appears in the 5+1D effective action and behaves like,

$$\int d^6x \frac{(\partial\Phi)^4}{|\Phi|^3}\,. \tag{4.4}$$

This term is of the same order of magnitude as (4.3) and its existence in the 5+1D effective action is suggested by M(atrix) theory. It would give the correct v^4/r^3 behavior for the potential between far away gravitons in M-theory compactified on $\mathbf{T}^4$. We will also see below how terms similar in structure to (4.4) are related to (4.3) by supersymmetry.

After dimensional reduction to 3+1D we obtain a term which behaves like

$$(\mathrm{Im}\tau)^{1/2} A^{1/2} \int d^4x \frac{(\partial\varphi)^4}{|\varphi|^3}\,. \tag{4.5}$$

This is valid when $\Phi_0 A \gg 1$. On the other hand, when $\Phi_0 A \ll 1$, $N = 4$ SYM with a coupling constant given by the combination τ is a good approximation, at low enough energies (around the scale of $\Phi_0 A^{1/2}$). In SYM, 1-loop effects can produce a term that behaves like (see [21]),

$$\int d^4x \frac{(\partial\varphi)^4}{|\varphi|^4}\,. \tag{4.6}$$

Note that this term contains no τ, and no A.

How can we interpolate between (4.3) and (4.6)?

The answer lies in the periodicity of σ. For any value of $\Phi_0 A$ the formula must be periodic in the 6th scalar σ, according to (4.1). Thus, we propose to write

$$\int d^4x (\partial\varphi)^4 \sum_{k\in\mathbb{Z}} \frac{1}{[\sum_{A=1}^5 |\varphi_A|^2 + (\sigma + 2k\pi(\mathrm{Im}\tau)^{-1/2}A^{-1/2})^2]^2}\,. \tag{4.7}$$

For small A we can keep only the term with $k = 0$ and recover (4.6). For large A we have to approximate the sum by an integral and we obtain

$$\begin{aligned}
&\sum_{k\in\mathbb{Z}} \frac{1}{[\sum_{A=1}^5 |\varphi_A|^2 + (\sigma + 2k\pi(\mathrm{Im}\tau)^{-1/2}A^{-1/2})^2]^2} \\
&\qquad \sim \int_{-\infty}^{\infty} \frac{dk}{[\sum_{A=1}^5 |\varphi_A|^2 + (\sigma + 2k\pi(\mathrm{Im}\tau)^{-1/2}A^{-1/2})^2]^2} \\
&\qquad = \frac{1}{4}(\mathrm{Im}\tau)^{1/2}A^{1/2} \frac{1}{(\sum_{A=1}^5 |\varphi_A|^2)^{3/2}}\,.
\end{aligned}$$

Thus we recover roughly (4.5). One can make a similar conjecture for the generalization of (4.3) by changing the power of the denominator in the denominator from 2 to 5/2 and modifying the numerator according to (4.3). It is also easy to see, by Poisson resummation, that the corrections to the integral fall off exponentially like (using (4.2)),

$$\exp\left\{-(\mathrm{Im}\tau)^{1/2}A^{1/2}(\sum_{A=1}^{5}|\varphi_A|^2)^{1/2}\}\right\} = e^{-\Phi_0 A},$$

and so are related to instantons made by strings wrapping the $\mathbf{T}^2$. There are no corrections which behave like Yang-Mills instantons, i.e. $e^{2\pi i\tau}$. The reason for this was explained in [21], in the SYM limit.

4.3. A derivation from 4+1D SYM

When we compactify the $(2,0)_{(N=2)}$ theory on $\mathbf{S}^1$ of radius L, we find a low-energy description of $U(1)^2$ SYM. When $\Phi_0 L^2 \ll 1$ and when the energies are much smaller than L^{-1}, the effective 4+1D SYM Lagrangian with $U(2)$ gauge group is a good approximation.

The moduli space is $\mathbb{R}^5/\mathbb{Z}_2$ and the term (3.1) implies that there is a term in the Lagrangian which is proportional to (we have switched to physical units),

$$g\epsilon_{ABCDE}\int d^5x\ \frac{1}{|\varphi|^5}\epsilon^{\alpha\beta\gamma\delta\mu}A_\mu\varphi^E\partial_\alpha\varphi^A\partial_\beta\varphi^B\partial_\gamma\varphi^C\partial_\delta\varphi^D\,.$$

This term can actually be seen as a 1-loop effect! Let us consider a loop of a charged gluino with 4 external legs of scalars and 1 external leg of a photon. Let the external momenta be

$$k_1, k_2, \cdots, k_5$$

The loop behaves as,

$$g^5\mathrm{tr}\{t^1t^2t^3t^4\}\int d^5p\ \mathrm{tr}\{\gamma_\mu\frac{1}{\not{p}-m}\cdot\frac{1}{\not{p}+\not{k}_1-m}\cdots\frac{1}{\not{p}+\not{k}_1+\cdots\not{k}_4-m}\}\,. \tag{4.8}$$

Here m is the mass of the gluino and is proportional to $g\varphi_0$. The coupling constant g is proportional to $\sqrt{L}$ (see appendix). The term with $\epsilon^{\alpha\beta\gamma\delta\mu}$ comes from expanding (4.8) in the $\not{k}_i$. We find

$$g^5\mathrm{tr}\{t^1t^2t^3t^4\}\mathrm{tr}\{\gamma_\mu\not{k}_1\not{k}_2\not{k}_3\not{k}_4\}m\int\frac{d^5p}{(p^2+m^2)^5}\sim g^5m^{-4}\epsilon^{ABCDE}\epsilon_{\alpha\beta\gamma\delta\mu}k_1^\alpha k_2^\beta k_3^\gamma k_4^\delta\,.$$

This is the behavior that we want. It would be interesting to check if a similar term appears in the low energy description of the M-theory on T^6 as a matrix model [27]-[48]. In a certain regime we can approximate by 6+1D Yang-Mills. For the $SU(2)$ case the moduli space is $\mathbb{R}^3/\mathbb{Z}_2$. A similar effect could generate term of the form below.

$$\int A\wedge F\wedge F\wedge\epsilon_{ABC}\frac{\phi^A d\phi^B\wedge d\phi^C}{|\phi|^3}\,.$$

After completion of this work, we have found out that such terms were indeed calculated in [51]. We are grateful to G. Thompson for pointing this out to us.

4.4. Component form

Let us see how to write the term (4.3) in an $N=1$ superfield notation. Let us take three chiral superfields, Φ and Φ^I $(I=1,2)$. We assume that

$$\Phi = \varphi_0 + \delta\varphi + i\sigma\,.$$

σ is the imaginary part of Φ and φ_0 is the VEV of the real part. Below, the index I of Φ^I is lowered and raised with the anti-symmetric ϵ_{IJ}.

Let us examine the following term:

$$I_1 = \frac{1}{32\pi}(\mathrm{Im}\tau)^{1/2}A^{1/2}\int d^4x d^4\theta \frac{1}{(\Phi\overline{\Phi}+\Phi^I\overline{\Phi}_I)^{3/2}}\bar{D}^{\dot{\alpha}}\overline{\Phi}^I D^\alpha\Phi_I\sigma^\mu_{\alpha\dot{\alpha}}\overline{\Phi}^J\partial_\mu\Phi_J + c.c. \quad (4.9)$$

We can expand

$$\begin{aligned}
&\int d^4x d^4\theta \frac{1}{(\Phi\overline{\Phi}+\Phi^I\overline{\Phi}_I)^{3/2}}\bar{D}^{\dot{\alpha}}\overline{\Phi}^I D^\alpha\Phi_I\sigma^\mu_{\alpha\dot{\alpha}}\overline{\Phi}^J\partial_\mu\Phi_J \\
&\quad = \frac{1}{{\varphi_0}^3}\int d^4x d^4\theta \bar{D}^{\dot{\alpha}}\overline{\Phi}^I D^\alpha\Phi_I\sigma^\mu_{\alpha\dot{\alpha}}\overline{\Phi}^J\partial_\mu\Phi_J \\
&\quad - \frac{3}{2{\varphi_0}^4}\int d^4x d^4\theta(\Phi+\overline{\Phi}-2\varphi_0)\bar{D}^{\dot{\alpha}}\overline{\Phi}^I D^\alpha\Phi_I\sigma^\mu_{\alpha\dot{\alpha}}\overline{\Phi}^J\partial_\mu\Phi_J + O(\frac{1}{{\varphi_0}^5})\,. \quad (4.10)
\end{aligned}$$

Let us denote

$$\begin{aligned}
I_2 &= \frac{i}{8{\varphi_0}^3}\int d^4x d^4\theta \bar{D}^{\dot{\alpha}}\overline{\Phi}^I D^\alpha\Phi_I\sigma^\mu_{\alpha\dot{\alpha}}\overline{\Phi}^J\partial_\mu\Phi_J + c.c.\,, \\
I_3 &= \frac{i}{8{\varphi_0}^4}\int d^4x d^4\theta\ (\Phi+\overline{\Phi}-2\varphi_0)\bar{D}^{\dot{\alpha}}\overline{\Phi}^I D^\alpha\Phi_I\sigma^\mu_{\alpha\dot{\alpha}}\overline{\Phi}^J\partial_\mu\Phi_J + c.c.\,, \qquad (4.11)
\end{aligned}$$

Let us check the bosonic part of I_1. We use $\widetilde{\varphi}$ and $\widetilde{\varphi}^I$ for the scalar components of Φ and Φ^I. We will expand in inverse powers of φ_0 and keep only leading terms.

It is easy to see that I_3 contains the term

$$\frac{1}{{\varphi_0}^4}\int d^4x\ \sigma\epsilon^{\alpha\beta\gamma\delta}\partial_\alpha\widetilde{\varphi}^I\partial_\beta\widetilde{\varphi}_I\partial_\gamma\overline{\widetilde{\varphi}}^J\partial_\delta\overline{\widetilde{\varphi}}_J \qquad (4.12)$$

At the order of $1/{\varphi_0}^3$ there are a few more terms that do not include Φ. They are listed below.

$$\begin{aligned}
J_1 &= \frac{1}{{\varphi_0}^3}\int d^4\theta\partial_\mu\Phi^I\partial^\mu\Phi^J\overline{\Phi}_I\overline{\Phi}_J + \frac{1}{{\varphi_0}^3}\int d^4\theta\partial_\mu\overline{\Phi}^I\partial^\mu\overline{\Phi}^J\Phi_I\Phi_J\,, \\
J_2 &= \frac{1}{{\varphi_0}^3}\int d^4\theta\partial_\mu\Phi^I\partial^\mu\overline{\Phi}_I\Phi^J\overline{\Phi}_J\,, \\
J_3 &= \frac{1}{{\varphi_0}^3}\int d^4\theta\partial_\mu\Phi^I\partial^\mu\overline{\Phi}^J\Phi_I\overline{\Phi}_J\,, \\
J_4 &= \frac{1}{{\varphi_0}^3}\int d^4\theta\sigma^\mu_{\alpha\dot{\alpha}}D^\alpha\Phi^I\bar{D}^{\dot{\alpha}}\overline{\Phi}_I\partial_\mu\Phi^J\overline{\Phi}_J\,, \qquad (4.13)
\end{aligned}$$

We now write down the bosonic terms of the above

$$\begin{aligned}
J_3 &= \frac{1}{{\varphi_0}^3}\int d^4x\,\{\tilde\varphi_I\partial^\mu\tilde\varphi^I\partial^\nu\bar{\tilde\varphi}_J\partial_\mu\partial_\nu\bar{\tilde\varphi}^J-\tilde\varphi_I\partial_\mu\partial_\nu\tilde\varphi^I\bar{\tilde\varphi}_J\partial^\mu\partial^\nu\bar{\tilde\varphi}^J\}\\
J_2 &= \frac{1}{2{\varphi_0}^3}\int d^4x\,\{-4\tilde\varphi_I\partial_\mu\partial_\nu\tilde\varphi_J\bar{\tilde\varphi}^I\partial^\mu\partial^\nu\bar{\tilde\varphi}^J+2\tilde\varphi_I\partial_\mu\partial_\nu\tilde\varphi_J\bar{\tilde\varphi}^J\partial^\mu\partial^\nu\bar{\tilde\varphi}^I\\
&\quad -2\tilde\varphi_I\partial_\mu\partial_\nu\tilde\varphi_J\partial^\nu\bar{\tilde\varphi}^I\partial^\mu\bar{\tilde\varphi}^J-3\tilde\varphi_I\tilde\varphi_J\partial_\mu\partial_\nu\bar{\tilde\varphi}^I\partial^\mu\partial^\nu\bar{\tilde\varphi}^J\\
&\quad +\tilde\varphi_I\partial_\mu\tilde\varphi^I\partial_\nu\bar{\tilde\varphi}_J\partial^\mu\partial^\nu\bar{\tilde\varphi}^J\}\\
J_1 &= \frac{1}{2{\varphi_0}^3}\int d^4x\,\{6\tilde\varphi_I\partial_\mu\partial_\nu\tilde\varphi_J\partial^\mu\bar{\tilde\varphi}^I\partial^\nu\bar{\tilde\varphi}^J+3\tilde\varphi_I\partial_\mu\partial_\nu\tilde\varphi_J\bar{\tilde\varphi}^J\partial^\mu\partial^\nu\bar{\tilde\varphi}^I\\
&\quad +3\tilde\varphi_I\partial_\mu\partial_\nu\tilde\varphi_J\bar{\tilde\varphi}^I\partial^\mu\partial^\nu\bar{\tilde\varphi}^J+\tilde\varphi_I\tilde\varphi_J\partial_\mu\partial_\nu\bar{\tilde\varphi}^I\partial^\mu\partial^\nu\bar{\tilde\varphi}^J\}\\
J_4 &= \frac{8}{{\varphi_0}^3}\int d^4x\,\{\partial_\mu\bar{\tilde\varphi}_I\partial_\nu\bar{\tilde\varphi}^I\partial^\mu\tilde\varphi_J\partial^\nu\tilde\varphi^J+\partial_\nu\bar{\tilde\varphi}_J\partial^\nu\bar{\tilde\varphi}^I\partial^\mu\tilde\varphi_I\partial_\mu\tilde\varphi^J\\
&\quad -2\bar{\tilde\varphi}_J\partial^\mu\bar{\tilde\varphi}^I\partial^\nu\tilde\varphi_I\partial_\mu\partial_\nu\tilde\varphi^J\}\,. \qquad (4.14)
\end{aligned}$$

We will now check which combination has the following symmetry which is part of $SO(5)$ and doesn't involve $\tilde\varphi$ and $\bar{\tilde\varphi}$,

$$\delta\tilde\varphi^I=\bar{\tilde\varphi}^I\,. \qquad (4.15)$$

We find

$$\begin{aligned}
\delta J_1 &= \frac{1}{{\varphi_0}^3}\int d^4x\,\{4\tilde\varphi_I\bar{\tilde\varphi}_J\partial_\mu\partial^\nu\bar{\tilde\varphi}^I\partial_\nu\partial^\mu\bar{\tilde\varphi}^J-4\tilde\varphi_I\partial_\mu\tilde\varphi^I\partial_\nu\tilde\varphi_J\partial^\mu\partial^\nu\bar{\tilde\varphi}^J\}\\
\delta J_2 &= \frac{1}{2{\varphi_0}^3}\int d^4x\,\{5\tilde\varphi_I\partial_\mu\bar{\tilde\varphi}^I\partial_\nu\bar{\tilde\varphi}_J\partial^\mu\partial^\nu\bar{\tilde\varphi}^J-3\tilde\varphi_I\bar{\tilde\varphi}_J\partial_\mu\partial_\nu\bar{\tilde\varphi}^I\partial^\mu\partial^\nu\bar{\tilde\varphi}^J\\
&\quad +8\tilde\varphi_I\partial_\nu\tilde\varphi^I\partial_\mu\tilde\varphi_J\partial^\mu\partial^\nu\bar{\tilde\varphi}^J\}\\
\delta J_3 &= \frac{1}{{\varphi_0}^3}\int d^4x\,\{2\tilde\varphi_I\partial_\mu\bar{\tilde\varphi}^I\partial_\nu\bar{\tilde\varphi}_J\partial^\mu\partial^\nu\bar{\tilde\varphi}^J+2\tilde\varphi_I\partial_\mu\tilde\varphi^I\partial_\nu\tilde\varphi_J\partial^\mu\partial^\nu\bar{\tilde\varphi}^J\}\\
\delta J_4 &= \frac{8}{{\varphi_0}^3}\int d^4x\,\{2\tilde\varphi_I\partial_\nu\bar{\tilde\varphi}_J\partial_\mu\bar{\tilde\varphi}^I\partial^\mu\partial^\nu\bar{\tilde\varphi}^J+2\tilde\varphi_I\bar{\tilde\varphi}_J\partial_\mu\partial_\nu\bar{\tilde\varphi}^I\partial^\mu\partial^\nu\bar{\tilde\varphi}^J\}\,. \qquad (4.16)
\end{aligned}$$

This puts some restrictions on the possible $1/{\Phi_0}^3$ term

$$\begin{aligned}
&{\varphi_0}^3\delta(C_1J_1+C_2J_2+C_3J_3+C_4J_4)\\
&\qquad =(4C_1-\frac{3}{2}C_2+16C_4)\tilde\varphi_I\bar{\tilde\varphi}_J\partial_\mu\partial^\nu\bar{\tilde\varphi}^I\partial_\nu\partial^\mu\bar{\tilde\varphi}^J\\
&\qquad\quad +(\frac{5}{2}C_2+2C_3+16C_4)\tilde\varphi_I\partial_\mu\bar{\tilde\varphi}^I\partial_\nu\bar{\tilde\varphi}_J\partial^\mu\partial^\nu\bar{\tilde\varphi}^J\\
&\qquad\quad +(-4C_1+4C_2+2C_3)\tilde\varphi_I\partial_\nu\tilde\varphi^I\partial_\mu\tilde\varphi_J\partial^\mu\partial^\nu\bar{\tilde\varphi}^J\,. \qquad (4.17)
\end{aligned}$$

We see that

$$\frac{5}{2}C_2+2C_3+16C_4=0\,,\qquad 4C_1-\frac{3}{2}C_2+16C_4=0\,.$$

Thus, we need to take the following $SO(4)$ invariant combination:

$$C(3J_1+8J_2-10J_3)+C'(4J_1+8J_3-J_4)\,,$$

where C, C' are undetermined. We have not checked if one can extend it to a supersymmetric and $SO(5)$ invariant combination by including interactions with Φ [22]. We thank Savdeep Sethi for discussions on this point.

5. Conserved quantities

We can check that the overall "center of mass" decouples. We can write it as a conservation equation for the total dissolved membrane charge (j_Z), total transverse momentum (j_Φ) and kinematical supersymmetry (j_Θ):

$$j_Z^{\alpha,\beta\gamma} = \frac{1}{2\pi}\sum_{i=1}^N H_i^{\alpha\beta\gamma}\,, \quad j_\Phi^{\alpha,A} = \frac{1}{2\pi}\sum_{i=1}^N \partial^\alpha \Phi_i^A\,, \quad j_\Theta^\alpha = \frac{1}{2\pi}\sum_{i=1}^N \Gamma^\alpha \Theta_i\,. \tag{5.1}$$

They are conserved simply because $\partial_\alpha j^\alpha$ gives the sum over i, j of the right hand sides of (3.1, 3.2, 3.8) but the summand is ij antisymmetric. The charges are defined as the integrals of the $\alpha = 0$ (lowered index) components

$$Z^{IJ} = \int \frac{d^5\sigma}{2\pi}\sum_{i=1}^N H_0^{i,IJ}, \quad P^A = \int \frac{d^5\sigma}{2\pi}\sum_{i=1}^N \partial_0 \Phi_i^A\,, \quad Q^{KIN} = \int \frac{d^5\sigma}{2\pi}\sum_{i=1}^N \Gamma_0 \Theta_i\,. \tag{5.2}$$

We use the terms "dissolved membranes" and "thin membranes" for membranes of M-theory with 0 or 1 directions transverse to the fivebranes, respectively. The thin membrane charge appears as a central charge in the supersymmetry algebra [20]. The reason is that $\{Q, \bar{Q}\}$ in M-theory contains momenta, twobrane and fivebrane charges. But in (2,0) theory, only the generators with $\widetilde{\Gamma}Q = Q$ i.e. $\bar{Q}\widetilde{\Gamma} = -\bar{Q}$ survive. So we see that $\{Q, \bar{Q}\}$ is a matrix anticommuting with $\widetilde{\Gamma}$ (i.e. containing an odd number of Greek indices). For momenta it means that only momenta inside the fivebrane worldvolume appear on RHS of supersymmetry algebra because Γ_μ anticommutes with $\widetilde{\Gamma}$ while the transverse Γ_A commutes with $\widetilde{\Gamma}$.

Only membrane charges contain $\Gamma_\mu \Gamma_A$ which anticommutes with $\widetilde{\Gamma}$ while $\Gamma_{\mu\nu}$ and Γ_{AB} commute with $\widetilde{\Gamma}$. This is an explanation why the thin membranes (looking like strings) with one direction transverse to the fivebrane occur on the RHS of the supersymmetry algebra. There are also 3-form central charges which appear with $\Gamma_{\mu\nu\sigma}\Gamma_A$ in the SUSY algebra. These correspond to tensor fluxes of the 3-form H (analogous to electric and magnetic fluxes in Yang-Mills theories). But let us return to the thin membranes. We should be able to find the corresponding current. The answer is (up to an overall normalization)

$$M_{\alpha,\beta}^A = -\frac{1}{12\pi}\epsilon_{\alpha\beta\gamma\delta\epsilon\zeta}\sum_{i=1}^N \partial^\gamma(\Phi_i^A H_i^{\delta\epsilon\zeta}) = \frac{1}{2\pi}\sum_{i=1}^N \partial^\gamma(H_{\alpha\beta\gamma}^i \Phi_i^A)\,. \tag{5.3}$$

The conservation law $\partial^\alpha M_{\alpha\beta}^A$ is a simple consequence of $\alpha\gamma$ anti-symmetry of $\epsilon_{\alpha\beta\gamma\delta\epsilon\zeta}$. It is also easy to see that for a configuration containing a membrane, the total integral

$\int d^5\sigma M^A_{0I} = W_I \cdot \Delta\Phi^A$ measures the membrane charge. Here W_I is the winding vector of the induced string and $\Delta\Phi^A$ is the asymptotic separation of the two fivebranes.

There must be also a current corresponding to the $SO(5)$ R-symmetry. It is given by

$$R^{AB}_\alpha = \frac{1}{2\pi}\sum_{i=1}^{N}\left(2\Phi_i^{[A}\partial_\alpha\Phi_i^{B]} - \frac{i}{2}\bar{\Theta}_i\Gamma_\alpha\Gamma^{AB}\Theta_i\right) + \text{corrections.} \tag{5.4}$$

It is also quite remarkable that the corrected equations conserve the stress energy tensor known from free theory. For the initial considerations, let us restrict our attention to the bosonic part of the stress tensor and choose the sign so that $T_{00} > 0$ i.e. $T^0_{\ 0} < 0$. Ignoring the requirement of the vanishing trace (i.e. without the second derivatives that we discuss below), the bosonic part of our stress tensor is given by

$$T^{try}_{\alpha\beta} = \frac{1}{2\pi}\sum_{i=1}^{N}\left(\frac{1}{4}H^i_{\alpha\gamma\delta}H_\beta^{i,\gamma\delta} + \partial_\alpha\Phi^i_A\partial_\beta\Phi^i_A - \frac{1}{2}\eta_{\alpha\beta}\partial_\gamma\Phi^i_A\partial^\gamma\Phi^i_A\right). \tag{5.5}$$

Note that the Φ part has nonzero trace. The divergence of this symmetric tensor can be written as

$$\partial^\alpha T^{try}_{\alpha\delta} = \frac{1}{2\pi}\sum_{i=1}^{N}\left(\frac{1}{2}H^i_{\delta\gamma\delta'}(\partial_\alpha H^{i,\alpha\gamma\delta'}) + (\Box\Phi^i_D)\partial_\delta\Phi^i_D\right). \tag{5.6}$$

If we substitute $\Box\Phi_D$ from (3.2) and $\partial_\alpha H^{i,\alpha\gamma\delta'}$ from (3.1) we obtain $\partial^\alpha T^{try}_{\alpha\delta} = 0$.

We should note one thing that could be confusing. In the M-theory containing N fivebranes, the stress tensor is not equal to zero but rather to[1]

$$T^M_{\alpha\beta} = -N\tau^{(5)}\eta_{\alpha\beta} + T_{\alpha\beta}\,, \tag{5.7}$$

where our $T_{\alpha\beta}$ is just a small correction to the infinite first term given by the tension of the fivebrane $\tau^{(5)}$. The first term is in the limit of (2,0) theory infinite because $\tau^{(5)}$ is of order l^{-6}_{Planck} and l_{Planck} is much smaller than a typical distance inside fivebranes studied by (2,0) theory. Nevertheless, gravity in this limit decouples and thus the "cosmological" term in (5.7) plays no role.

5.1. Traceless stress tensor and supercurrent

In this subsection, we exhibit a traceless version of the stress tensor and the supercurrent. We will use the adjective "traceless" both for the supercurrent J_α and the stress tensor $T_{\alpha\beta}$ which means that

$$\Gamma^\alpha J_\alpha = 0, \quad T^\alpha_\alpha = 0\,. \tag{5.8}$$

The supercurrent has positive chirality $(\widetilde{\Gamma} - 1)J_\alpha = 0$ – it means that the total supercharges have positive chirality as well. We will also require continuity for stress tensor and supercurrent.

$$\partial^\alpha J_\alpha = 0, \quad \partial^\alpha T_{\alpha\beta} = 0\,. \tag{5.9}$$

[1]The minus sign in (5.7) is because our choice of the spacelike metric and $T_{00} > 0$.

Our definition of the stress tensor will be finally

$$
\begin{aligned}
T_{\alpha\beta} = \frac{1}{2\pi} \Bigg\{ & \sum_{i=1}^{N} \frac{1}{4} H^i_{\alpha\gamma\delta} H_\beta^{i,\gamma\delta} + \\
& + \sum_{i=1}^{N} \frac{3}{5} \left(\partial_\alpha \Phi^i_A \partial_\beta \Phi^i_A - \frac{1}{6} \eta_{\alpha\beta} \partial_\gamma \Phi^i_A \partial^\gamma \Phi^i_A \right) - \frac{2}{5} \Phi^i_A \left(\partial_\alpha \partial_\beta - \frac{1}{6} \eta_{\alpha\beta} \Box \right) \Phi^i_A + \\
& + \sum_{i=1}^{N} \frac{(-i)}{2} \bar{\Theta}^i \left(\Gamma_{(\alpha} \partial_{\beta)} - \frac{1}{6} \eta_{\alpha\beta} \not{\partial} \right) \Theta^i \Bigg\} . \qquad (5.10)
\end{aligned}
$$

We fixed a normalization for H, Φ, Θ in this equation. The factors $1/6$ inside the parentheses guarantee the tracelessness while the relative factor $-3/2$ between the parentheses ensures vanishing of the dangerous terms in $\partial^\alpha T_{\alpha\beta}$ which cannot be expressed from the equations of motion, namely $\partial^\alpha \Phi \partial_{\alpha\beta} \Phi$. The H^2 part of the stress tensor is traceless identically. An explicit calculation shows that for the divergence of the stress tensor we get

$$
\begin{aligned}
\partial^\alpha T_{\alpha\beta} = \frac{1}{2\pi} \sum_{i=1}^{N} \Bigg[& \frac{1}{2} (\partial^\alpha H^i_{\alpha\gamma\delta}) H_\beta^{i,\gamma\delta} + \frac{2}{3} (\partial_\beta \Phi^i_A)(\Box \Phi^A_i) - \frac{1}{3} \Phi^i_A (\partial_\beta \Box \Phi^A_i) + \\
& + \frac{7i}{12} (\partial_\beta \bar{\Theta}^i)(\not{\partial} \Theta^i) - \frac{i}{4} \bar{\Theta}^i \Gamma_\beta \Box \Theta^i - \frac{i}{6} \bar{\Theta}^i \partial_\beta \not{\partial} \Theta^i \Bigg] . \qquad (5.11)
\end{aligned}
$$

A similar approach can be used for the supercurrent as well. Here also the $H\Theta$ part is traceless identically while for the other parts it is ensured by the $1/6$ factors. The relative factor $-3/2$ between the parentheses is again chosen to cancel the dangerous $\partial^\alpha \Phi \partial_\alpha \Theta$ terms in $\partial^\alpha J_\alpha$. Note that the structure of J_α mimics the form of $T_{\alpha\beta}$.

$$
\begin{aligned}
J_\alpha = & \frac{1}{24\pi} \sum_{i=1}^{N} H_i^{\beta\gamma\delta} \Gamma_{\beta\gamma\delta} \Gamma_\alpha \Theta^i + \\
& + \frac{1}{\pi} \sum_{i=1}^{N} \left[\frac{3}{5} \left(\partial_\alpha \Phi^i_A - \frac{1}{6} \Gamma_\alpha \not{\partial} \Phi^i_A \right) \Gamma_A \Theta^i - \frac{2}{5} \Phi^i_A \left(\partial_\alpha - \frac{1}{6} \Gamma_\alpha \not{\partial} \right) \Gamma^A \Theta^i \right] . \qquad (5.12)
\end{aligned}
$$

We can compute also a similar continuity equation for the supercurrent as we did for the stress tensor. The result is

$$
\begin{aligned}
\partial^\alpha J_\alpha = & \frac{1}{4\pi} \partial_\alpha H^{\alpha\beta\gamma} \Gamma_{\beta\gamma} \Theta + \frac{1}{24\pi} H^{\beta\gamma\delta} \Gamma_{\beta\gamma\delta} \not{\partial} \Theta \\
& + \frac{1}{2\pi} \left[(\Box \Phi_A) \Gamma^A \Theta - \frac{1}{3} \not{\partial} \Phi_A \Gamma^A (\not{\partial} \Theta) - \frac{2}{3} \Phi_A \Gamma^A (\Box \Theta) \right] . \qquad (5.13)
\end{aligned}
$$

Using the equations of motion and the integration by parts, the Hamiltonian and the total supercharge defined as

$$
\mathcal{H} = \int d^5\sigma T_{00} \, , \qquad Q = \int d^5\sigma J_0 \qquad (5.14)
$$

can be easily expressed as

$$
\mathcal{H} = \frac{1}{2\pi} \int d^5\sigma \left(\frac{1}{2} \left(\Pi^2 + (\nabla \Phi)^2 \right) + \frac{1}{12} H_{KLM} H^{KLM} + \frac{(-i)}{2} \bar{\Theta} \Gamma^J \partial_J \Theta \right) . \qquad (5.15)
$$

(we use conventions with $[\Pi(x),\Phi(y)] = -2\pi i\delta^{(5)}(x-y)$ and $\{\Theta^s(x),\bar\Theta_{s'}(y)\} = \pi((1+\tilde\Gamma)\Gamma_0)^s{}_{s'}\delta^{(5)}(x-y))$ and

$$Q = \frac{1}{2\pi}\int d^5\sigma\left(\frac{1}{6}H^{IJK}\Gamma_{IJK}\Gamma_0\Theta - \partial_\beta\Phi_A\Gamma^\beta\Gamma^0\Gamma^A\Theta\right). \tag{5.16}$$

For convenience, we can also easily compute

$$\bar Q = \frac{1}{2\pi}\int d^5\sigma\left(\frac{1}{6}\bar\Theta\Gamma^0\Gamma_{IJK}H^{IJK} + \bar\Theta\Gamma^0\Gamma^\beta\Gamma^A\partial_\beta\Phi_A\right) \tag{5.17}$$

using a simple identity

$$\overline{(\Theta\Gamma_{\mu_1}\ldots\Gamma_{\mu_N})} = (-1)^N\bar\Theta\Gamma_{\mu_N}\ldots\Gamma_{\mu_1}\,. \tag{5.18}$$

Now we can consider the supersymmetry transformation. A variation of a field F will be written as

$$\delta F = [\bar\epsilon Q, F] = [\bar Q\epsilon, F]\,. \tag{5.19}$$

Note that $\bar\epsilon Q$ is an antihermitean operator because the components of Q or ϵ are hermitean anticommuting operators or numbers, respectively. Using the canonical commutation relations we can easily compute the variations of the fields

$$\delta\Theta = -[\Theta,\bar Q\epsilon] = \left(\frac{1}{6}\Gamma_{IJK}H^{IJK} + \Gamma^\beta\Gamma^A\partial_\beta\Phi_A\right)\epsilon\,. \tag{5.20}$$

This agrees with the transformations written before. This, together with the normalization of $\{Q,\bar Q\}$ is how we determined the relative coefficients. Similarly

$$\delta\Phi_A = [\bar\epsilon Q,\Phi_A] = \bar\epsilon[Q,\Phi_A] = \bar\epsilon[\frac{1}{2\pi}\int d^5\sigma\Pi_B\Gamma^B\Theta,\Phi_A] = -i\bar\epsilon\Gamma_A\Theta\,, \tag{5.21}$$

which agrees with previous definitions. A similar but more tedious calculation gives us

$$\delta H_{IJK} = \frac{i}{2}\bar\epsilon\cdot\epsilon_{0J'K'IJK}\Gamma^{I'J'K'}\Gamma_0\partial_{I'}\Theta\,, \tag{5.22}$$

which also agrees with the previous definition.

Let us summarize some formulas that are useful in understanding the commutator of two supersymmetry transformations:

$$\delta f = [\bar\epsilon Q, f] = [\bar Q\epsilon, f], \qquad \partial_\alpha f = -i[P_\alpha, f], \qquad P^0 = \mathcal{H} > 0 \tag{5.23}$$

$$\{Q_s,\bar Q^{s'}\} = -2P^\mu((1+\tilde\Gamma)/2\cdot\Gamma_\mu)_s{}^{s'} + \text{thin} \tag{5.24}$$

$$\Rightarrow \delta Q = -[Q,\bar Q\epsilon] = 2P^\mu\Gamma_\mu\epsilon + \text{thin} \tag{5.25}$$

$$\delta J_\alpha = -2T_{\alpha\beta}\Gamma^\beta\epsilon + \text{thin} \tag{5.26}$$

$$(\delta_1\delta_2 - \delta_2\delta_1)f = [\bar\epsilon_1 Q,[\bar\epsilon_2 Q,f]] - [\bar\epsilon_2 Q,[\bar\epsilon_1 Q,f]] = [[\bar\epsilon_1 Q,\bar\epsilon_2 Q],f] \tag{5.27}$$

$$= [\bar\epsilon_1^s\{Q_s,\bar Q^{s'}\}\epsilon_{2,s'},f] = -2[P^\mu\bar\epsilon_1\Gamma_\mu\epsilon_2,f] + \text{thin} \tag{5.28}$$

$$= -2i(\bar\epsilon_1\Gamma^\mu\epsilon_2)\partial_\mu f + \text{thin} \tag{5.29}$$

6. Speculations over a fundamental formulation

In this section we would like to speculate on whether a fundamental formulation of the (2,0) theory can be constructed from the equations we discussed above. We warn the reader in advance that this section could cause some gritting of teeth! Of course, the correction terms are not renormalizable if treated as "fundamental" but let us go on, anyway. Perhaps some hidden symmetry makes them renormalizable after all?

The model has the following virtues:

- there are absolutely no new fields. We use only N copies of the field strength H_{MNP}, five scalars Φ_A and the 16 component fermion Θ. Because of that, restriction to N copies for distant fivebranes is almost manifest.

- the string current automatically satisfies the quantization condition as a right winding number. This is related to the fact that our current is automatically conserved (obeys the continuity equation) which is necessary to allow us to insert it to equation $dH = J$ – and it has the correct dimension $mass^4$.

- the total charge (sum over $1...N$) vanishes. The string (membrane connecting fivebranes) brings correctly minus source to one fivebrane and plus source to the other which agrees with the fact that the oriented membrane is outgoing from one fivebrane and incoming to another fivebrane – and with $e_i - e_j$ roots of $U(N)$

- the model is symmetric with respect to the correct Hořava-Witten symmetry [34] that accompanies the reflection $\Phi^i_A \to -\Phi^i_A$ by changing sign of C_{MNP} (i.e. of H_{MNP}).

- string states are given by strange configuration of fivebranes so that the vector of direction between two Φ's draws whole S^4 (surface of ball in $\mathbb{R}^5$) if one moves in the 4 transverse directions of the string.

- $U(N)$ is not manifest, it arises due to the string states – perhaps in analogy with the way enhanced symmetries appear in string theory because of D-brane bound states.

What does a string look like? It is a solution constant in the time and in one spatial direction, with a given asymptotical value of $\Delta\Phi = |\Phi^i - \Phi^j|$ in infinity. We can show that such solution will have typical size of order $\Delta\Phi^{-1/2}$ in order to minimize the tension (energy per unit of length of the string).

The value of $\partial\Phi$ is of order $\Delta\Phi/s$, integral of its square over the volume s^4 is of order $(s\Delta\Phi)^2$. On the contrary, such a topological charge makes the field H to behave like $1/r^3$ where r is the distance from the center of the solution. Therefore H inside the solution is of order $1/s^3$ which means that the contribution of H^2 to the tension is of order $s^4/s^6 = 1/s^2$. The total tension $(s\Delta\Phi)^2 + 1/s^2$ is minimal for $s = (\Delta\Phi)^{-1/2}$ and the tension is therefore of order $\Delta\Phi$. The field Φ tries to shrink the solution while

H attempts to blow it up. In the next section we will describe the solution more concretely.

7. String-like solution of (2,0) theory

We will try to describe the string-like solution of the bosonic part of the equations, considering only the topological term of dH and the corresponding term in $\Box\Phi$ equation. The following discussion is somewhat reminiscent of a related discussion in [23] for the effect of higher order derivative terms on monopole solutions in $N = 2$ Yang-Mills but our setting is different.

7.1. A rough picture

Our solution will be constant in σ^0, σ^5 coordinates but it will depend on the four coordinates $\sigma^1, \sigma^2, \sigma^3, \sigma^4$. We are looking for a solution that minimizes the energy. If the size of the solution in these four directions is of order s, then the "electric" field, going like $1/r^3$, is of order $1/s^3$ inside the solution and therefore the integral $d^4\sigma(H^2)$, proportional to the tension, is of order $s^4/(s^3)^2$.

On the contrary, for the asymptotic separation $\Delta\Phi$ quantities $\partial\Phi$ are of order $\Delta\Phi/s$ inside the typical size of the solution and therefore the contribution to the tension $d^4\sigma(\partial\Phi)^2$ is of order $s^4(\Delta\Phi/s)^2$.

Minimizing the total tension $1/s^2 + s^2\Delta\Phi^2$ we get the typical size $s = (\Delta\Phi)^{-1/2}$ and the tension of order $\Delta\Phi$. In this reasoning, we used the energy known from the free theory because the bosonic part of the interacting stress energy tensor equals the free stress energy tensor. The fact that the solution corresponds to the interacting theory (and not to the free theory) is related to the different constraint for $(dH)_{IJKL}$.

7.2. The Ansatz

We will consider $N = 2$ case of the (2,0) theory, describing two fivebranes. Our solution will correspond to the membrane stretched between these two fivebranes. Denoting by (1) and (2) the two fivebranes, we will assume $\Phi_{(1)} = -\Phi_{(2)}$, $H_{(1)} = -H_{(2)}$ and we denote $\Phi_{(1)}$ and $H_{(1)}$ simply as Φ and H.

Our solution will be invariant under $SO(4)_D$ rotating spacetime and the transverse directions together. The variable

$$r = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2}$$

measures the distance from the center of the solution. We choose the asymptotic separation to be in the 10th direction and we denote it as

$$\Phi^{10}(\infty) = \frac{1}{2}\Delta\Phi\,.$$

Now there is an arbitrariness in the identification of the coordinates $1,2,3,4$ and $6,7,8,9$. So there is in fact a moduli space of classical solutions, corresponding to the chosen identification of these coordinates. According to our Ansatz, the solution will be determined in the terms of the three functions.

$$\Phi^{I+5} = \sigma^I f_1(r),\ I = 1,2,3,4 \quad \Phi^{10} = f_2(r)\,, \quad B_{05} = f_3(r).$$

We set the other components of $B_{\mu\nu}$ to zero and define H as the anti-self-dual part of dB,

$$H_{\alpha\beta\gamma} = \frac{3}{2}\partial_{[\alpha}B_{\beta\gamma]} - \text{dual expression}\,.$$

It means that $H_{05I} = 1/2 \cdot \partial_I f_3$ and the selfduality says

$$H_{051} = -H_{234}, \quad H_{052} = H_{134}, \quad H_{053} = -H_{124}, \quad H_{054} = H_{123}.$$

Now we can go through the equations. dH equations for $1,2,3,4$ determines $-4\partial_{[1}H_{234]} = \partial_I H_{05I} = \frac{1}{2}\Delta f_3$ where we used $\Delta = \Box$ because of the static character. Therefore dH equation says

$$\Delta f_3 = -8c_1 \frac{f_1^3}{(f_2^2 + r^2 f_1^2)^{5/2}}(-rf_1 f_2' + f_1 f_2 + r f_1' f_2).$$

The three factors f_1 arose from $\partial_2\Phi_7$, $\partial_3\Phi_8$, $\partial_4\Phi_9$, we calculated everything at $\sigma^{1,2,3,4} = (r,0,0,0)$. At this point, only $EABCD = 10,6789$ and $6,10,789$ from ϵ symbol contributed. Here Δ always denotes the spherically symmetric part of the laplacian in 4 dimensions, i.e.

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{3}{r}\frac{\partial}{\partial r}\,.$$

Similarly, we get hopefully two equations from $\Box\Phi$. For Φ^{10} (in the direction of asymptotic separation), we seem to get

$$\Delta f_2 = 6\frac{c_2}{2}\partial_1(-B_{05})\frac{\epsilon^{7,8,9,10,6}}{(f_2^2 + r^2 f_1^2)^{5/2}} r f_1^4\,.$$

Similarly, for the four other components we have

$$\Delta(rf_1) = -3c_2 f_3' \frac{f_2 f_1^3}{(f_2^2 + r^2 f_1^2)^{5/2}}\,.$$

7.3. Numerical solution, tension and speculations

The functions f_1, f_2, f_3 are all even, therefore their derivatives are equal to zero for $r = 0$. The value of $f_2(0)$ finally determines $f_2(\infty)$ which we interpret as $\Delta\Phi/2$. The value of $f_1(0)$ must be fixed to achieve a good behavior at infinity and $f_3(0)$ has no physical meaning, because only derivatives of $f_3 = B_{05}$ enter the equations.

We can calculate the tension and we can compare the result with the BPS formula. If we understand our equations just as some low energy approximation, there should be no

JHEP05(1998)009

reasons to expect that the calculated tension will be precise, because the approximation breaks down at the core.

The tension expected from SYM theory is something like

$$M_W/L_5 = \Delta\Phi^{SYM} \cdot g/L_5 = \sqrt{2\pi/L_5}\Delta\Phi^{SYM} = \sqrt{2\pi}\Delta\Phi^{(2,0)}\,.$$

We just used simple formula for W boson masses, W bosons are string wound around 5th direction and the Φ fields of SYM and (2,0) are related by $\sqrt{L_5}$ ratio as well.

The tension from our (2,0) theory is just twice (the same contribution from two fivebranes) the integral

$$2\int d^4\sigma \frac{1}{4\pi}\left(H_{05I}^2 + (\partial_I \Phi^A)^2\right).$$

Because of the spherical symmetry, we can replace $\int d^4\sigma$ by $\int_0^\infty dr \cdot 2\pi^2 r^3$. Work is in progress.

8. Discussion

Recently, a prescription for answering questions about the large N limit of the (2,0) theory has been proposed [25]. In particular, the low-energy effective description for a single 5-brane separated from N 5-branes has been deduced [25]. The topological term that we have discussed is, of course, manifestly there. This is because a 5-brane probe in an $AdS_7 \times S^4$ feels the 4-form flux on S^4 and and this will induce the anomalous dH term.

What does M(atrix) theory have to say about non-linear corrections to the low-energy of the (2,0) theory? This is a two-sided question as the (2,0) theory is a M(atrix) model for M-theory on $\mathbf{T}^4$ [3, 4] and has a M(atrix) model of its own [7, 8].

In order to be able to apply our discussion of the uncompactified 5+1D (2,0) theory to the M(atrix) model for M-theory on $\mathbf{T}^4$ we need to be in a regime such that the VEV of the tensor multiplet is much larger than the size of $\hat{\mathbf{T}}^5$. This means that for a scattering process of two gravitons in M-theory on $\mathbf{T}^4$ the distance between the gravitons must remain much larger than the compactification scale which we assume is of the order of the 11D Planck scale. In this regime we expect the potential to behave as v^4/r^3 (in analogy with v^4/r^7 in 11D). Thus, things would work nicely if there were a term

$$\frac{(\partial\Phi)^4}{|\Phi|^3} \tag{8.1}$$

in the effective low-energy description in 5+1D. In the large N limit, the existence of this term has been observed in [25]. The term (8.1) will also be the leading term in the amplitude for a low-energy scattering of two massless particles in the (2,0) theory. It should thus be possible to calculate it from the M(atrix) model of the (2,0) theory, with a VEV turned on.

It is also interesting to ask whether a term like (8.1) is renormalized or not. An analysis which addresses such a question in 0+1D will appear in [24]. Perhaps a similar analysis in 5+1D would settle this question.

Acknowledgments

We are grateful to Micha Berkooz, Rami Entin, Savdeep Sethi, Zuzana Saková, Herman Verlinde, Edward Witten, and especially Tom Banks for valuable discussions. The work of L. Motl was supported in part by the DOE under grant number DE-FG02-96ER40559. The work of O. Ganor was supported in part by the DOE under the grant number DE-FG02-91ER40671.

A. Formulas for SUSY transformations

In this text, we will use the $SO(10,1)$ formalism for spinors, inherited from the M-theory containing N fivebranes, and the space-like metric (in 5, 6 and 11 dimensions)

$$\begin{aligned} \eta_{\mu\nu} &= \mathrm{diag}(-++++++++++)\,, \quad \mu,\nu = 0,1,\ldots 10. \\ ds^2 &= \eta_{\alpha\beta}dx^\alpha dx^\beta = -dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 + dx_5^2\,. \end{aligned} \tag{A.1}$$

A.1. SUSY transformation

The SUSY transformations of the free tensor multiplet in 5+1D is given by

$$\begin{aligned} \delta H_{\alpha\beta\gamma} &= -\frac{i}{2}\bar\epsilon\Gamma_\delta\Gamma_{\alpha\beta\gamma}\partial^\delta\Theta = -3i\bar\epsilon\Gamma_{[\alpha\beta}\partial_{\gamma]}\Theta + \frac{i}{2}\bar\epsilon\Gamma_{\alpha\beta\gamma}\Gamma_\delta\partial^\delta\Theta \\ \delta\Phi_A &= -i\bar\epsilon\Gamma_A\Theta \\ \delta\Theta &= (\frac{1}{12}H_{\alpha\beta\gamma}\Gamma^{\alpha\beta\gamma} + \Gamma^\alpha\partial_\alpha\Phi_A\Gamma^A)\epsilon\,. \end{aligned}$$

Since we are dealing with corrections to the low-energy equations of motion, it is important to keep terms which vanish by the equations of motion. The SUSY commutators are thus given by

$$\begin{aligned} (\delta_1\delta_2 - \delta_2\delta_1)H_{\alpha\beta\gamma} &= -2i(\bar\epsilon_1\Gamma^\mu\epsilon_2)\partial_\mu H_{\alpha\beta\gamma} \\ &\quad -\frac{i}{2}\epsilon_{[\alpha\beta}{}^{\delta\alpha'\beta'\gamma'}\bar\epsilon_1\Gamma_{\gamma]}\epsilon_2\partial_{[\delta}H_{\alpha'\beta'\gamma']} + 4i(\bar\epsilon_1\Gamma^\delta\epsilon_2)\partial_{[\delta}H_{\alpha\beta\gamma]} \\ (\delta_1\delta_2 - \delta_2\delta_1)\Theta &= -2i(\bar\epsilon_1\Gamma^\mu\epsilon_2)\partial_\mu\Theta \\ &\quad -\frac{i}{24}\left\{18(\bar\epsilon_2\Gamma_\mu\epsilon_1)\Gamma^\mu - 6(\bar\epsilon_2\Gamma_\mu\Gamma_A\epsilon_1)\Gamma^\mu\Gamma^A\right\}\Gamma_\beta\partial^\beta\Theta \\ (\delta_1\delta_2 - \delta_2\delta_1)\Phi_A &= -2i(\bar\epsilon_1\Gamma^\mu\epsilon_2)\partial_\mu\Phi_A\,. \end{aligned}$$

The equations of motion transform according to,

$$\delta\left(\Gamma_\delta\partial^\delta\Theta\right) = \frac{1}{6}\Gamma^{\delta\alpha\beta\gamma}\epsilon\partial_{[\delta}H_{\alpha\beta\gamma]} + \Gamma^A\epsilon\partial_\alpha\partial^\alpha\Phi_A\,,$$

$$\delta\left(\partial_{[\mu} H_{\alpha\beta\gamma]}\right) = \frac{i}{2}\bar{\epsilon}\Gamma_{[\alpha\beta\gamma}\partial_{\mu]}\left(\Gamma_\delta \partial^\delta \Theta\right),$$
$$\delta\left(\partial_\mu \partial^\mu \Phi_A\right) = -i\bar{\epsilon}\Gamma_A\left(\Gamma_{\delta'}\partial^{\delta'}\right)\Gamma_\delta\partial^\delta\Theta = -i\bar{\epsilon}\Gamma_A \Box\Theta\,.$$

B. Quantization

The quantization of the free tensor multiplet was discussed at length in [26]. There is no problem with the fermions Θ and bosons Φ^A, but the tensor field is self-dual and thus has to be quantized similarly to a chiral boson in 1+1D. This means that we second-quantize a free tensor field without any self-duality constraints and then set to zero all the oscillators with self-dual polarizations.

The analogy with chiral bosons is made more explicit if we compactify on $\mathbf{T}^4$ and take the low-energy limit we we can neglect Kaluza-Klein states. We obtain a 1+1D conformal theory. This theory is described by compact chiral bosons on a $(3,3)$ lattice. This is the lattice of fluxes on $\mathbf{T}^4$. For $\mathbf{T}^4$ which is a product of four circles with radii L_i $(i = 1\ldots 4)$, we get 3 non-chiral compact bosons with radii

$$\frac{L_1L_2}{L_3L_4},\ \frac{L_1L_3}{L_2L_4},\ \frac{L_1L_4}{L_2L_3}.$$

Of course, in 1+1D, T-duality can replace each radius R with $1/R$ and thus $SL(4,\mathbb{Z})$ invariance is preserved.

If we further compactify on $\mathbf{T}^5$ the zero modes will be described by quantum mechanics on $\mathbf{T}^{10}$, where $\mathbf{T}^{10}$ is the unit cell of the lattice of fluxes.

B.1. Commutators

Let us write down the commutation relations.

We want to reproduce the equations of motion by the Heisenberg equations

$$\partial_0(L) = i[\mathcal{H}, L] \quad \text{where} \quad \mathcal{H} = \int d^5\sigma T_{00}\,. \tag{B.1}$$

We should be allowed to substitute H, Φ, Θ for the operator L. In the following text we will use indices $I, J, K, \ldots$ for the spatial coordinates inside the fivebrane. We will keep the spacelike metric and the convention

$$\epsilon_{12345} = \epsilon^{12345} = 1\,. \tag{B.2}$$

We have the equations $H = -*H$ and $dH = 0$. Among the fifteen equations for the vanishing four-form $dH = 0$ we find ten equations with index 0. These will be satisfied as the Heisenberg equations (B.1). Remaining five equations with space-like indices will only play a role of some constraints that are necessary for consistent quantization as we will see. Let us take the example of equations of motion for $(dH)_{0345}$.

$$0 = \partial_0 H_{345} - \partial_3 H_{450} + \partial_4 H_{503} - \partial_5 H_{034} = \partial_0 H_{345} + \partial_3 H_{123} + \partial_4 H_{124} + \partial_5 H_{125}\,. \tag{B.3}$$

It means that we should have the commutator

$$i[\mathcal{H}, H_{345}(\sigma')] = -\partial_I^{(\sigma')} H_{12I}(\sigma')\,, \tag{B.4}$$

where the important part of hamiltonian is

$$\mathcal{H}_H = \frac{1}{8\pi}\int d^5\sigma H_{0IJ} H_0{}^{IJ} = \int d^5\sigma \frac{1}{24\pi} H_{KLM} H^{KLM}\,. \tag{B.5}$$

But it is straightforward to see that the relation (B.4) will be satisfied if the commutator of H's will be

$$[H_{IJK}(\sigma), H_{LMN}(\sigma')] = -6\pi i \partial_{[I}^{(\sigma)} \delta^{(5)}(\sigma - \sigma')\epsilon_{JK]LMN}\,. \tag{B.6}$$

What does all this mean for the particles of the H field? Let us study Fourier modes of H's with $\pm p_I$ where $p_I = (0,0,0,0,p)$. Then we can see that $H_{125}(p) = H_{125}(-p)^\dagger$ is a dual variable to $H_{345}(-p) = H_{345}(p)^\dagger$ and similarly for two other pairs which we get using cyclic permutations $12, 34 \to 23, 14 \to 31, 24$. So totally we have three physical polarizations of the tensor particle (which is of course the same number like that of polarizations of photon in $4+1$ dimensional gauge theory).

We can also easily see from (B.6) that the p-momentum modes of variables that do not contain index "5", namely $H_{123}, H_{124}, H_{134}, H_{234}$ commute with everything. They (more precisely their ∂_5 derivatives) exactly correspond to the components of dH, namely

$$(dH)_{1235}, (dH)_{1245}, (dH)_{1345}, (dH)_{2345} \tag{B.7}$$

that we keep to vanish as the constraint part of $dH = 0$. Let us just note that $(*_5 dH)_I = 0$ contains *four* conditions only because $d(dH) = 0$ is satisfied identically. Anyhow, there are no quantum mechanical variables coming from the components of $(dH)_I$. The variables dH are the generators of the two-form gauge invariance

$$B_{IJ} \mapsto B_{IJ} + \partial_I \lambda_J - \partial_J \lambda_I\,. \tag{B.8}$$

Note that for $\lambda_I = \partial_I \phi$ we get a trivial transformation of B's which is the counterpart of the identity $d(dH) = 0$.

But what about the zero modes, the integrals of H_{IJK} over the five-dimensional space? These are the ten fluxes that should be quantized, i.e. they should belong to a lattice. In the 4+1 dimensional SYM theory they appear as four electric and six magnetic fluxes. In the matrix model of M-theory on T^4 these ten variables are interpreted as four compact momenta and six transverse membrane charges.

The fact that "unpaired" degrees of freedom are restricted to a lattice is an old story. For instance, in the bosonic formulation of the heterotic string in 1+1 dimensions we have 16 left-moving (hermitean) bosons ("anti-self-dual field strengths") α^i, $i = 1,\dots,16$ with commutation relations

$$[\alpha^i(\sigma), \alpha^j(\sigma')] = i\delta'(\sigma - \sigma')\delta^{ij}\,. \tag{B.9}$$

After combining them to Fourier modes

$$\alpha^i(\sigma) = \sqrt{\frac{2}{\pi}} \sum_{n\in\mathbb{Z}} \alpha^i_n e^{-2i\sigma n} \qquad \Leftrightarrow \qquad \alpha^i_n = \frac{1}{\sqrt{2\pi}} \int_0^\pi \alpha^i(\sigma) e^{2i\sigma n} d\sigma\,, \tag{B.10}$$

we get relations

$$[\alpha^i_m, \alpha^j_n] = m\delta_{m+n}\delta^{ij}\,, \quad (\alpha^i_m)^\dagger = \alpha^i_{-m} \tag{B.11}$$

and we can interpret α^i_n and α^i_{-n} for $n > 0$ as annihilation and creation operators respectively. The modes α^i_0 are then restricted to belong to a selfdual lattice. Roughly speaking, α^i_0 equals the total momentum and it equals to the total winding vector due to selfduality – but these two must belong to mutually dual lattices. The lattice must be even in order for the operator

$$L =: \frac{1}{2} \sum_{n\in\mathbb{Z}} \alpha^i_{-m}\alpha^i_m :=: \frac{1}{4} \int_0^\pi \alpha(\sigma)^2 d\sigma : \tag{B.12}$$

to have integer eigenvalues. We see that the 480 ground level states $|0\rangle_{\alpha^i_0}$ with $(\alpha^i_0)^2 = 2$ give the same value $L = 1$ as the sixteen lowest excited states $\alpha^i_{-1}|0\rangle_{\alpha_0=0}$. These combine to the perfect number 496 of the states.

B.2. Correspondence with Super Yang Mills

We will use the normalization of the gauge theory with Lagrangian and covariant derivative as follows

$$\mathcal{L} = -\frac{1}{4g^2} F^{\mu\nu}F_{\mu\nu}\,, \qquad D_\alpha = \partial_\alpha + iA_\alpha\,. \tag{B.13}$$

The hamiltonian for the $U(1)$ theory then can be written as $(i, j = 1, 2, 3, 4)$

$$\mathcal{H}_{SYM} = \frac{1}{2g^2} \int d^4\sigma \left[\sum_i (E_i)^2 + \sum_{i<j} (F_{ij})^2 \right]. \tag{B.14}$$

Let us consider compactification on a rectangular T^5 (the generalization for other tori is straightforward) of volume $V = L_1L_2L_3L_4L_5$. We should get (B.14) from our hamiltonian. Let us write $d^5\sigma$ as $L_5 d^4\sigma$ (we suppose that the fields are constant in the extra fifth direction).

$$\mathcal{H}^{(2,0)}_{SYM} = \frac{L_5}{4\pi} \int d^4\sigma \sum_{I<J<K} (H_{IJK})^2\,. \tag{B.15}$$

So it is obvious that we must identify (up to signs) $F_{\alpha\beta}$ with $H_{\alpha\beta 5} \cdot g\sqrt{L_5/(2\pi)}$ e.g.

$$H_{234} = \frac{E_1}{g\sqrt{L_5/(2\pi)}}\,, \quad H_{125} = \frac{F_{12}}{g\sqrt{L_5/(2\pi)}}\,. \tag{B.16}$$

To change A_i of the SYM theory by a constant, we must take the phase ϕ of the gauge transformation to be a linear function of coordinates. But it should change by a multiple of 2π after we go around a circle. Thus

$$\phi = \frac{2\pi n_i}{L_i}\sigma_i\,, \quad A_i \to A_i + \frac{2\pi n_i}{L_i}\,. \tag{B.17}$$

The dual variable to the average value of A_i is the integral of E_i/g^2. We just showed that the average value of A_i lives on a circle with radius and therefore $L_1L_2L_3L_4 \cdot E_i/g^2$ belongs to the lattice with spacing L_i. Similarly, we can obtain a nonzero magnetic flux from the configuration (A_i can change only by a multiple of the quantum in (B.17))

$$A_i = \frac{2\pi n_{ij}}{L_i} \cdot \frac{\sigma_j}{L_j}\,, \tag{B.18}$$

which gives the magnetic field

$$F_{ij} = \frac{2\pi n_{ij}}{L_iL_j}. \tag{B.19}$$

Therefore for the spacings of the average values of E_i, F_{ij} we have

$$\Delta E_i = \frac{g^2 L_i}{L_1L_2L_3L_4}, \quad \Delta F_{ij} = \frac{2\pi}{L_iL_j}. \tag{B.20}$$

Looking at (B.16) we can write for the averages of H's e.g.

$$\Delta H_{234} = \frac{gL_1}{L_1L_2L_3L_4\sqrt{L_5/(2\pi)}}\,, \quad \Delta H_{125} = \frac{2\pi}{L_1L_2g\sqrt{L_5/(2\pi)}}\,, \tag{B.21}$$

which can be extended to a six-dimensionally covariant form only using the following precise relation between the coupling constant and the circumference L_5

$$g = \sqrt{2\pi L_5}\,, \tag{B.22}$$

giving us the final answer for the spacing

$$\Delta H_{IJK} = \frac{2\pi}{L_IL_JL_K}\,. \tag{B.23}$$

The formula (B.23) can be also written as

$$\frac{1}{6}\oint H_{IJK}dV^{IJK} \in 2\pi\cdot\mathbb{Z}\,, \tag{B.24}$$

or (using antiselfduality) as

$$\Delta\int d^5\sigma H^{0IJ} = 2\pi L^I L^J\,, \tag{B.25}$$

in accord with the interpretation of H as the current of dissolved membranes (the integral in (B.25) is the total membrane charge).

B.3. Normalization of the current

We can also work out the value of c_1 in (3.1). Let us write this equation for $\alpha\beta\gamma\delta = 1234$.

$$\partial_{[1} H_{234]} = \frac{1}{4}(\partial_1 H_{234} - \partial_2 H_{341} + \partial_3 H_{412} - \partial_4 H_{123}) = \frac{1}{4}\partial_\alpha H^{05\alpha} = J_{1234}\,. \tag{B.26}$$

We see from (B.16) that

$$J_{1234} = \frac{1}{4g\sqrt{L_5/(2\pi)}}\sum_{i=1}^{4}\partial_i E_i. \tag{B.27}$$

The integral of $\partial_i E_i$ should be an integer multiple of g^2 (in these conventions) and because of (B.27), the integral of J_{1234} should be an integer multiple of $\pi/2$ which was the way we determined the coefficient in (3.1).

C. Identities

C.1. Identities for gamma matrices

$$\begin{aligned}
\Gamma^\alpha\Gamma^\beta &= \Gamma^{\alpha\beta} + \eta^{\alpha\beta} && \text{(C.1)}\\
\Gamma^{\alpha'}\Gamma^{\beta'\gamma'} &= \Gamma^{\alpha'\beta'\gamma'} + \Gamma^{\gamma'}\eta^{\beta'\alpha'} - \Gamma^{\beta'}\eta^{\gamma'\alpha'}\,, && \text{(C.2)}\\
\Gamma^{\beta'\gamma'}\Gamma^{\alpha'} &= \Gamma^{\alpha'\beta'\gamma'} - \Gamma^{\gamma'}\eta^{\beta'\alpha'} + \Gamma^{\beta'}\eta^{\gamma'\alpha'}\,, && \text{(C.3)}
\end{aligned}$$

$$\Gamma^{\alpha'}\Gamma^{\beta'\gamma'} + \Gamma^{\beta'\gamma'}\Gamma^{\alpha'} = 2\Gamma^{\alpha'\beta'\gamma'} \tag{C.4}$$

$$\Gamma^{\alpha'}\Gamma_{\alpha\beta} - \Gamma_{\alpha\beta}\Gamma^{\alpha'} = 4\delta^{\alpha'}_{[\alpha}\Gamma_{\beta]} \tag{C.5}$$

$$\Gamma^\delta\Gamma^{\alpha\beta\gamma} = \Gamma^{\delta\alpha\beta\gamma} + 3\eta^{\delta[\alpha}\Gamma^{\beta\gamma]} \tag{C.6}$$

$$\Gamma^\delta\Gamma^{\alpha\beta\gamma} + \Gamma^{\alpha\beta\gamma}\Gamma^\delta = 6\eta^{\delta[\alpha}\Gamma^{\beta\gamma]} \tag{C.7}$$

$$\Gamma^{\alpha'\beta'\gamma'}\Gamma_{\alpha\beta} - \Gamma_{\alpha\beta}\Gamma^{\alpha'\beta'\gamma'} = 12\Gamma^{\delta'[\beta'\gamma'}\delta^{\alpha']}_{[\alpha}\eta_{\beta]\delta'} \tag{C.8}$$

$$\Gamma^{\alpha'\beta'\gamma'}\Gamma_{\alpha\beta} + \Gamma_{\alpha\beta}\Gamma^{\alpha'\beta'\gamma'} = -12\delta^{[\alpha'}_{[\alpha}\delta^{\beta'}_{\beta]}\Gamma^{\gamma']} + 2\Gamma^{\alpha'\beta'\gamma'}{}_{\alpha\beta} \tag{C.9}$$

$$\widetilde{\Gamma}\Gamma_{\mu_1\mu_2\cdots\mu_k} = (-1)^{k(k+1)/2}\frac{1}{(6-k)!}\epsilon_{\mu_1\mu_2\cdots\mu_k}{}^{\nu_1\nu_2\cdots\nu_{6-k}}\Gamma_{\nu_1\nu_2\cdots\nu_{6-k}} \tag{C.10}$$

$$\epsilon_{\mu_1\mu_2\cdots\mu_k}{}^{\nu_1\nu_2\cdots\nu_{6-k}} = (-1)^k\epsilon^{\nu_1\nu_2\cdots\nu_{6-k}}{}_{\mu_1\mu_2\cdots\mu_k} \tag{C.11}$$

$$\epsilon_{\mu_1\cdots\mu_{6-k}}{}^{\nu_1\cdots\nu_k}\Gamma^{\mu_1\cdots\mu_{6-k}} = (-1)^{k(k-1)/2}(6-k)!\cdot\widetilde{\Gamma}\Gamma^{\nu_1\cdots\nu_k} \tag{C.12}$$

$$\widetilde{\Gamma}^2 = +1 \tag{C.13}$$

$$
\begin{aligned}
\Gamma^{\alpha\beta}\Gamma_\beta &= 5\Gamma^\alpha && \text{(C.14)}\\
\Gamma^{\alpha}\Gamma_\alpha &= 6\mathbf{I}, && \text{(C.15)}\\
\Gamma^{\alpha}\Gamma_\mu\Gamma_\alpha &= -4\Gamma_\mu, && \text{(C.16)}\\
\Gamma^{\alpha}\Gamma_{\mu_1\mu_2\cdots\mu_k}\Gamma_\alpha &= (-1)^k(6-2k)\Gamma_{\mu_1\mu_2\cdots\mu_k}, && \text{(C.17)}\\
\Gamma^{\alpha_1\cdots\alpha_l}\Gamma_{\alpha_1\cdots\alpha_l} &= \frac{6!}{(6-l)!}(-1)^{l(l-1)/2}\mathbf{I}, && \text{(C.18)}\\
\Gamma^{\alpha_1\cdots\alpha_l}\Gamma_\mu\Gamma_{\alpha_1\cdots\alpha_l} &= (-1)^{l(l+1)/2}\frac{(6-2l)5!}{(6-l)!}\Gamma_\mu\,. && \text{(C.19)}
\end{aligned}
$$

$$
\begin{aligned}
\Gamma^{\alpha\beta}\Gamma_{\alpha\beta} &= -30\mathbf{I}, & \Gamma^{\alpha\beta\gamma}\Gamma_{\alpha\beta\gamma} &= -120\mathbf{I},\\
\Gamma^{\alpha\beta}\Gamma_\mu\Gamma_{\alpha\beta} &= -10\Gamma_\mu, & \Gamma^{\alpha\beta\gamma}\Gamma_\mu\Gamma_{\alpha\beta\gamma} &= 0,\\
\Gamma^{\alpha\beta}\Gamma_{\mu\nu}\Gamma_{\alpha\beta} &= 2\Gamma_{\mu\nu}\,, & \Gamma^{\alpha\beta\gamma}\Gamma_{\mu\nu}\Gamma_{\alpha\beta\gamma} &= 24\Gamma_{\mu\nu}\,,\\
\Gamma^{\alpha\beta}\Gamma_{\mu\nu\sigma}\Gamma_{\alpha\beta} &= 6\Gamma_{\mu\nu\sigma}\,, & \Gamma^{\alpha\beta\gamma}\Gamma_{\mu\nu\sigma}\Gamma_{\alpha\beta\gamma} &= 0.
\end{aligned}
\tag{C.20}
$$

Derivation for last equations:

$$
\begin{aligned}
\Gamma^{\alpha\beta\gamma}\Gamma_{\mu\nu(\sigma)}\Gamma_{\alpha\beta\gamma} &= (\Gamma^{\beta\gamma}\Gamma^\alpha + \Gamma^\gamma\eta^{\beta\alpha} - \Gamma^\beta\eta^{\gamma\alpha})\Gamma_{\mu\nu(\sigma)}\Gamma_{\alpha\beta\gamma} = \Gamma^{\beta\gamma}\Gamma^\alpha\Gamma_{\mu\nu(\sigma)}\Gamma_{\alpha\beta\gamma}\\
&= \Gamma^{\beta\gamma}\Gamma^\alpha\Gamma_{\mu\nu(\sigma)}(\Gamma_\alpha\Gamma_{\beta\gamma} + \Gamma_\beta\eta_{\alpha\gamma} - \Gamma_\gamma\eta_{\alpha\beta})\\
&= \Gamma^{\beta\gamma}\Gamma^\alpha\Gamma_{\mu\nu(\sigma)}\Gamma_\alpha\Gamma_{\beta\gamma} + 2\Gamma^{\gamma\beta}\Gamma_\beta\Gamma_{\mu\nu(\sigma)}\Gamma_\gamma.
\end{aligned}
\tag{C.21}
$$

$$
\begin{aligned}
\Gamma^A\Gamma_A &= 5\mathbf{I}\,,\\
\Gamma^A\Gamma_B\Gamma_A &= -3\Gamma_B\,,\\
\Gamma^A\Gamma_{BC}\Gamma_A &= \Gamma_{BC}\,,\\
\Gamma^A\Gamma_{B_1B_2\cdots B_k}\Gamma_A &= (-1)^k(5-2k)\Gamma_{B_1B_2\cdots B_k}\,.
\end{aligned}
\tag{C.22}
$$

$$
\mathrm{tr}\{\Gamma^{\mu_1\mu_2\cdots\mu_k}\Gamma_{\nu_1\nu_2\cdots\nu_k}\} = 32k!(-1)^{\frac{k(k-1)}{2}}\delta^{[\mu_1}_{[\nu_1}\delta^{\mu_2}_{\nu_2}\cdots\delta^{\mu_k]}_{\nu_k]}. \tag{C.23}
$$

$$
\begin{aligned}
&\mathrm{tr}\{\Gamma^{\mu_1\cdots\mu_k}\Gamma^{A_1\cdots A_l}\Gamma_{\nu_1\cdots\nu_k}\Gamma_{B_1\cdots B_l}\}\\
&\qquad = 32k!l!(-1)^{\frac{(k+l)(k+l-1)}{2}}\delta^{[\mu_1}_{[\nu_1}\delta^{\mu_2}_{\nu_2}\cdots\delta^{\mu_k]}_{\nu_k]}\delta^{[A_1}_{[B_1}\delta^{A_2}_{B_2}\cdots\delta^{A_k]}_{B_k]}\,.
\end{aligned}
\tag{C.24}
$$

C.2. Fierz rearrangments

We need an identity of the form

$$
M_{mn} \equiv (\epsilon_1)_m(\bar\epsilon_2)_n = \Big(\sum_{k=0}^{6}\sum_{l=0}^{2} C_{\mu_1\cdots\mu_kA_1\cdots A_l}\Gamma^{\mu_1\cdots\mu_k}\Gamma^{A_1\cdots A_l}\Big)_{mn}\,. \tag{C.25}
$$

($l \le 2$ because $\Gamma^{01\ldots10} = 1$.) We then get:

$$
C_{\mu_1\cdots\mu_kA_1\cdots A_l} = \frac{(-1)^{\frac{(k+l)(k+l-1)}{2}}}{32k!l!}\mathrm{tr}\{M\Gamma_{\mu_1\cdots\mu_k}\Gamma_{A_1\cdots A_l}\}\,. \tag{C.26}
$$

Now we take

$$
\epsilon_2 = -\widetilde\Gamma\epsilon_2\,,\qquad \epsilon_1 = -\widetilde\Gamma\epsilon_1\,. \tag{C.27}
$$

and rearrange $M = \epsilon_1\bar\epsilon_2$. Now $\tilde\Gamma M = -M = -M\tilde\Gamma$ and we see that only terms with odd k survive.

$$
\begin{aligned}
M &\equiv \epsilon_1\bar\epsilon_2 \\
&= \left(-\frac{(\bar\epsilon_2\Gamma_\mu\epsilon_1)}{32}\Gamma^\mu + \frac{(\bar\epsilon_2\Gamma_\mu\Gamma_A\epsilon_1)}{32}\Gamma^\mu\Gamma^A + \frac{(\bar\epsilon_2\Gamma_\mu\Gamma_{AB}\epsilon_1)}{64}\Gamma^\mu\Gamma^{AB}\right)(1+\tilde\Gamma) \\
&\quad +\frac{1}{192}(\bar\epsilon_2\Gamma_{\mu\nu\sigma}\epsilon_1)\Gamma^{\mu\nu\sigma} - \frac{1}{192}(\bar\epsilon_2\Gamma_{\mu\nu\sigma}\Gamma_A\epsilon_1)\Gamma^{\mu\nu\sigma}\Gamma^A \\
&\quad -\frac{1}{384}(\bar\epsilon_2\Gamma_{\mu\nu\sigma}\Gamma_{AB}\epsilon_1)\Gamma^{\mu\nu\sigma}\Gamma^{AB}\,. \qquad \text{(C.28)} \\
N &\equiv \epsilon_1\bar\epsilon_2 - \epsilon_2\bar\epsilon_1 \\
&= \left(-\frac{1}{16}(\bar\epsilon_2\Gamma_\mu\epsilon_1)\Gamma^\mu + \frac{1}{16}(\bar\epsilon_2\Gamma_\mu\Gamma_A\epsilon_1)\Gamma^\mu\Gamma^A\right)(1+\tilde\Gamma) \\
&\quad -\frac{1}{192}(\bar\epsilon_2\Gamma_{\mu\nu\sigma}\Gamma_{AB}\epsilon_1)\Gamma^{\mu\nu\sigma}\Gamma^{AB}\,. \qquad \text{(C.29)} \\
L &\equiv \epsilon_1\bar\epsilon_2 + \epsilon_2\bar\epsilon_1 \\
&= \frac{1}{32}(\bar\epsilon_2\Gamma_\mu\Gamma_{AB}\epsilon_1)\Gamma^\mu\Gamma^{AB}(1+\tilde\Gamma) \\
&\quad +\frac{1}{96}(\bar\epsilon_2\Gamma_{\mu\nu\sigma}\epsilon_1)\Gamma^{\mu\nu\sigma} - \frac{1}{96}(\bar\epsilon_2\Gamma_{\mu\nu\sigma}\Gamma_A\epsilon_1)\Gamma^{\mu\nu\sigma}\Gamma^A\,. \qquad \text{(C.30)}
\end{aligned}
$$

where we have used, e.g.

$$\bar\epsilon_2\Gamma_{\mu\nu\sigma}\epsilon_1 = \bar\epsilon_1\Gamma_{\mu\nu\sigma}\epsilon_2\,. \qquad \text{(C.31)}$$

For opposite chirality spinors we have to replace $\tilde\Gamma$ by $-\tilde\Gamma$.

For, perhaps, future use, we will also calculate this for $M = \psi_1\bar\epsilon_2$ with

$$\epsilon_2 = -\tilde\Gamma\epsilon_2, \qquad \psi_1 = \tilde\Gamma\psi_1\,. \qquad \text{(C.32)}$$

$$
\begin{aligned}
M \equiv \psi_1\bar\epsilon_2 &= \left(-\frac{1}{32}(\bar\epsilon_2\psi_1)\mathbf{I} - \frac{1}{32}(\bar\epsilon_2\Gamma_A\psi_1)\Gamma^A + \frac{1}{64}(\bar\epsilon_2\Gamma_{AB}\psi_1)\Gamma^{AB}\right. \\
&\quad +\frac{1}{64}(\bar\epsilon_2\Gamma_{\mu\nu}\psi_1)\Gamma^{\mu\nu} + \frac{1}{64}(\bar\epsilon_2\Gamma_{\mu\nu}\Gamma_A\psi_1)\Gamma^{\mu\nu}\Gamma^A \\
&\quad \left.-\frac{1}{128}(\bar\epsilon_2\Gamma_{\mu\nu}\Gamma_{AB}\psi_1)\Gamma^{\mu\nu}\Gamma^{AB}\right)(1+\tilde\Gamma)\,. \qquad \text{(C.33)}
\end{aligned}
$$

we also need

$$
\begin{aligned}
N \equiv \Gamma_{\alpha\beta}\psi_1\bar\epsilon_2\Gamma^{\alpha\beta} &= \left(\frac{15}{16}(\bar\epsilon_2\psi_1)\mathbf{I} + \frac{15}{16}(\bar\epsilon_2\Gamma_A\psi_1)\Gamma^A - \frac{15}{32}(\bar\epsilon_2\Gamma_{AB}\psi_1)\Gamma^{AB}\right. \\
&\quad +\frac{1}{32}(\bar\epsilon_2\Gamma_{\mu\nu}\psi_1)\Gamma^{\mu\nu} + \frac{1}{32}(\bar\epsilon_2\Gamma_{\mu\nu}\Gamma_A\psi_1)\Gamma^{\mu\nu}\Gamma^A \\
&\quad \left.-\frac{1}{64}(\bar\epsilon_2\Gamma_{\mu\nu}\Gamma_{AB}\psi_1)\Gamma^{\mu\nu}\Gamma^{AB}\right)(1+\tilde\Gamma). \qquad \text{(C.34)}
\end{aligned}
$$

and

$$K \equiv \Gamma_A\psi_1\bar\epsilon_2\Gamma^A = \left(-\frac{5}{32}(\bar\epsilon_2\psi_1)\mathbf{I} + \frac{3}{32}(\bar\epsilon_2\Gamma_A\psi_1)\Gamma^A + \frac{1}{64}(\bar\epsilon_2\Gamma_{AB}\psi_1)\Gamma^{AB}\right.$$

$$+\frac{5}{64}(\bar{\epsilon}_2\Gamma_{\mu\nu}\psi_1)\Gamma^{\mu\nu} - \frac{3}{64}(\bar{\epsilon}_2\Gamma_{\mu\nu}\Gamma_A\psi_1)\Gamma^{\mu\nu}\Gamma^A$$
$$-\frac{1}{128}(\bar{\epsilon}_2\Gamma_{\mu\nu}\Gamma_{AB}\psi_1)\Gamma^{\mu\nu}\Gamma^{AB}\Big)(1+\tilde{\Gamma})\,. \tag{C.35}$$

C.3. Few notes about $\mathfrak{spin}(5,1)$

We use the eleven-dimensional language for the spinors. But nevertheless one could be confused by some elementary facts concerning the reality condition for the spinor $(4,4)$ of $\mathfrak{spin}(5,1)\times\mathfrak{spin}(5)$. The spinor representation 4 of $\mathfrak{spin}(5)$ is quaternionic (pseudoreal). Therefore $(4,4)$ of $\mathfrak{spin}(5)\times\mathfrak{spin}(5)$ is a real 16-dimensional representation. But one might think that spinor 4 of $\mathfrak{spin}(5,1)$ is *complex* so that we cannot impose a reality condition for the $(4,4)$ representation.

But of course, this is not the case. The spinor representation 4 of $\mathfrak{spin}(5,1)$ is *quaternionic* as well since the algebra $\mathfrak{spin}(5,1)$ can be understood also as $\mathfrak{sl}(2,\mathbb{H})$ of 2×2 quaternionic matrices with unit determinant of its 8×8 real form. This has the right dimension

$$4\cdot 4-1=15=\frac{6\cdot 5}{2\cdot 1}\,. \tag{C.36}$$

In the language of complex matrices, there is a matrix j_1 so that

$$(j_1)^2=-1\,,\qquad j_1M_1=\bar{M}_1j_1 \tag{C.37}$$

for all 4×4 complex matrices M_1 of $\mathfrak{spin}(5,1)$. Of course, for the 4×4 matrices M_2 in $\mathfrak{spin}(5)$ there is also such a matrix j_2 that

$$(j_2)^2=-1\,,\qquad j_2M_2=\bar{M}_2j_2\,. \tag{C.38}$$

An explicit form for the equations (C.37)-(C.38) is built from 2×2 blocks

$$j_1=\begin{pmatrix}\circ & 1\\ -1 & \circ\end{pmatrix}\,,\qquad M_1=\begin{pmatrix}\alpha & \beta\\ -\bar{\beta} & \bar{\alpha}\end{pmatrix}\,. \tag{C.39}$$

In the $(4,4)$ representation of $\mathfrak{spin}(5,1)\times\mathfrak{spin}(5)$ the matrices are given by $M=M_1\otimes M_2$ and therefore we can define a matrix j that shows that M is equivalent to a real matrix

$$j=j_1\otimes j_2\,,\quad j^2=1\,,\quad jM=j_1M_1\otimes j_2M_2=\bar{M}_1j_1\otimes\bar{M}_2j_2=\bar{M}j\,. \tag{C.40}$$

The algebra $\mathfrak{spin}(5,1)$ is quite exceptional between the other forms of $\mathfrak{spin}(6)$. The algebra $\mathfrak{so}(6)$ is isomorphic to $\mathfrak{su}(4)$, algebra $\mathfrak{so}(4,2)$ to $\mathfrak{su}(2,2)$ and algebra $\mathfrak{so}(3,3)$ to $\mathfrak{su}(3,1)$. The other form of $\mathfrak{su}(4)$ isomorphic to $\mathfrak{so}(5,1)$ is sometimes denoted $\mathfrak{su}^*(4)$ but now we can write it as $\mathfrak{sl}(2,\mathbb{H})$ as well (the generators are 2×2 quaternionic matrices with vanishing real part of the trace). From the notation $\mathfrak{sl}(2,\mathbb{H})$ it is also obvious that $\mathfrak{u}(2,\mathbb{H})=\mathfrak{usp}(4)$ forms a subgroup (which is isomorphic to $\mathfrak{so}(5)$).

References

[1] E. Witten, *Some comments on string dynamics*, hep-th/9507121, published in *Future perspectives in string theory*, Los Angeles 1995, p. 501-523.

[2] A. Strominger, *Open P-branes*, *Phys. Lett.* **B 383** (1996) 44 [hep-th/9512059].

[3] M. Rozali, *Matrix theory and U-duality in seven dimensions* *Phys. Lett.* **B 400** (1997) 260, hep-th/9702136.

[4] M. Berkooz, M. Rozali, N. Seiberg, *On transverse fivebranes in M(atrix) theory on T^5*, *Phys. Lett.* **B 408** (1997) 105 [hep-th/9704089].

[5] M. Berkooz, M. Rozali, *String dualities from matrix theory*, *Nucl. Phys.* **B 516** (1998) 229 [hep-th/9705175].

[6] T. Banks, W. Fischler, S.H. Shenker, L. Susskind, *M theory as a matrix model: a conjecture*, *Phys. Rev.* **D 55** (1997) 5112 [hep-th/9610043].

[7] O. Aharony, M. Berkooz, S. Kachru, N. Seiberg, E. Silverstein, *Matrix description of interacting theories in six dimensions*, *Adv. Theor. Math. Phys.* **1** (1998) 148 [hep-th/9707079].

[8] E. Witten, *On the conformal field theory of the Higgs branch*, *JHEP* **07** (1997) 003 [hep-th/9707093].

[9] D. Lowe, *$E_8 \times E_8$ small instantons in matrix theory*, hep-th/9709015.

[10] O. Aharony, M. Berkooz, S. Kachru, E. Silverstein, *Matrix description of (1,0) theories in six dimensions*, *Phys. Lett.* **B 420** (1998) 55 [hep-th/9709118].

[11] O. Aharony, M. Berkooz, N. Seiberg, *Light-cone description of (2,0) superconformal theories in six dimensions*, hep-th/9712117.

[12] M. Aganagic, C. Popescu, and J.H. Schwarz, *D-brane actions with local kappa symmetry*, *Phys. Lett.* **B 393** (1997) 311 [hep-th/9610249].

[13] M. Aganagic, J. Park, C. Popescu, and J.H. Schwarz, *World-volume action of the M theory five-brane*, *Nucl. Phys.* **B 496** (1997) 191 [hep-th/9701166].

[14] C. Vafa and E. Witten, *A strong coupling test of S-duality*, *Nucl. Phys.* **B 431** (1994) 3 [hep-th/9408074].

[15] M.J. Duff, J.T. Liu and R. Minasian, *Eleven dimensional origin of string/string duality: a one loop test*, *Nucl. Phys.* **B 452** (1995) 261 [hep-th/9506126].

[16] A. Hanany and E. Witten, *Type II B superstrings, BPS monopoles, and three-dimensional gauge dynamics*, *Nucl. Phys.* **B 492** (1997) 152 [hep-th/9611230].

JHEP05(1998)009

[17] P.S. Howe, E. Sezgin and P.C. West, *Covariant field equations of the M-theory five-brane* *Phys. Lett.* **B 399** (97) 49 [hep-th/9702008];
P.S. Howe, E. Lambert and P.C. West, *The selfdual string soliton*, *Nucl. Phys.* **B 515** (1998) 203 [hep-th/9709014];
E. Lambert and P.C. West, *Gauge fields and M five-brane dynamics*, hep-th/9712040.

[18] N. Seiberg, *Notes on theories with 16 supercharges*, hep-th/9705117.

[19] U. Lindstrom, F. Gonzalez-Rey, M. Roček, R. von Unge *On $N = 2$ low-energy effective actions*, *Phys. Lett.* **B 388** (1996) 581 [hep-th/9607089].

[20] N. Seiberg and E. Witten, *Comments on string dynamics in six-dimensions*, *Nucl. Phys.* **B 471** (1996) 121 [hep-th/9603003].

[21] M. Dine and N. Seiberg, *Comments on higher derivative operators in some SUSY field theories*, *Phys. Lett.* **B 409** (1997) 239 [hep-th/9705057].

[22] work in progress.

[23] G. Chalmers, M. Roček, and R. von Unge, *Quantum corrections to monopoles*, hep-th/9801043.

[24] S. Paban, S. Sethi and M. Stern, to appear.

[25] J. Maldacena, *The large N limit of superconformal field theories and supergravity*, hep-th/9711200.

[26] E. Witten, *Five-brane effective action in M-theory* *J. Geom. Phys.* **22** (1997) 103 [hep-th/9610234].

[27] N. Seiberg, *Why is the matrix model correct?*, *Phys. Rev. Lett.* **79** (1997) 3577 [hep-th/9710009].

[28] O. Ganor, *Compactification of tensionless string theories*, hep-th/9607092.

[29] B. de Wit, J. Hoppe, H. Nicolai, *On the quantum mechanics of supermembranes*, *Nucl. Phys.* **B 305** (1988) 545.

[30] P.K. Townsend, *D-branes from M-branes* *Phys. Lett.* **B 373** (1996) 68 [hep-th/9512062].

[31] L. Susskind, *Another conjecture about M(atrix) theory*, hep-th/9704080.

[32] A. Connes, M.R. Douglas, A. Schwarz, *Noncommutative geometry and matrix theory: compactification on tori*, hep-th/9711162.

[33] L. Motl, L. Susskind, *Finite N heterotic matrix models and discrete light cone quantization*, hep-th/9708083.

[34] P. Hořava, E. Witten, *Heterotic and type I string dynamics from eleven dimensions* *Nucl. Phys.* **B 460** (1996) 506 [hep-th/9510209].

[35] U. Danielsson, G. Ferretti, *The heterotic life of the D-particle*, *Int. J. Mod. Phys.* **A 12** (1997) 4581 [hep-th/9610082];
S. Kachru, E. Silverstein, *On gauge bosons in the matrix model approach to M theory*, *Phys. Lett.* **B 396** (1997) 70 [hep-th/9612162];
L. Motl, *Quaternions and M(atrix) theory in spaces with boundaries*, hep-th/9612198;
N. Kim, S.-J. Rey, *M(atrix) theory on an orbifold and twisted membrane*, *Nucl. Phys.* **B 504** (1997) 189 [hep-th/9701139];
D. Lowe, *Bound states of type I' D-particles and enhanced gauge symmetry*, *Nucl. Phys.* **B 01** (1997) 134 [hep-th/9702006];
T. Banks, N. Seiberg, E. Silverstein, *Zero and one-dimensional probes with N=8 supersymmetry*, *Phys. Lett.* **B 401** (1997) 30 [hep-th/9703052];
T. Banks, L. Motl, *Heterotic strings from matrices*, *JHEP* **12** (1997) 004 [hep-th/9703218];
P. Hořava, *Matrix theory and heterotic strings on tori*, *Nucl. Phys.* **B 505** (1997) 84 [hep-th/9705055].

[36] J. Polchinski, E. Witten, *Evidence for heterotic – type I string duality*, *Nucl. Phys.* **B 460** (1996) 525 [hep-th/9510169].

[37] J. Maldacena, L. Susskind, *D-branes and fat black holes*, *Nucl. Phys.* **B 475** (1996) 679 [hep-th/9604042];
S. Das, S. Mathur, *Comparing decay rates for black holes and D-branes*, *Nucl. Phys.* **B 478** (1996) 561 [hep-th/9606185];
G. Moore, R. Dijkgraaf, E. Verlinde, H. Verlinde, *Elliptic Genera of Symmetric Products and Second Quantized Strings*, *Comm. Math. Phys.* **185** (1997) 197 [hep-th/9608096].

[38] L. Motl, *Proposals on nonperturbative superstring interactions*, hep-th/9701025.

[39] T. Banks, N. Seiberg, *Strings from matrices*, *Nucl. Phys.* **B 497** (1997) 41 [hep-th/9702187].

[40] R. Dijkgraaf, E. Verlinde, H. Verlinde, *Matrix string theory*, *Nucl. Phys.* **B 500** (1997) 43 [hep-th/9703030].

[41] M.B. Green, M. Gutperle, P. Vanhove, *One loop in eleven dimensions*, *Phys. Lett.* **B 409** (1997) 177 [hep-th/9706175].

[42] M.B. Green and M. Gutperle, λ^{16} *and related terms in M-theory on* T^2, *Phys. Lett.* **B 421** (1998) 149 [hep-th/9710151].

[43] N. Seiberg, *Why is the matrix model correct?*, *Phys. Rev. Lett.* **79** (1997) 3577 [hep-th/9710009].

[44] A. Sen, *D0 branes on* T^n *and matrix theory*, hep-th/9709220.

[45] N. Seiberg, *Matrix description of M-theory on* T^5 *and* T^5/Z_2, *Phys. Lett.* **B 408** (1997) 98 [hep-th/9705221].

[46] I. Brunner and A. Karch, *Matrix description of M-theory on T^6*, *Phys. Lett.* **B 416** (1998) 67 [hep-th/9707259].

[47] A. Losev, G. Moore and S.L. Shatashvili, *M&m's*, hep-th/9707250.

[48] A. Hanany and G. Lifschytz, *M(atrix) theory on T^6 and a m(atrix) theory description of KK monopoles*, hep-th/9708037.

[49] J. Polchinski and P. Pouliot, *Membrane scattering with M-momentum transfer*, *Phys. Rev.* **D 56** (1997) 6601 [hep-th/9704029].

[50] T. Banks, W.Fischler, N. Seiberg and L. Susskind, *Instantons, scale invariance and Lorentz invariance in matrix theory*, *Phys. Lett.* **B 408** (1997) 111 [hep-th/9705190].

[51] C. Boulahouache, G. Thompson, *One loop effects in various dimensions and D-branes*, hep-th/9801083.